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ON PROPORTIONATE AND TRUTHFUL INTERNATIONAL ALLIANCE CONTRIBUTIONS: AN ANALYSIS OF INCENTIVE COMPATIBLE COST SHARING MECHANISMS TO BURDEN SHARING

THESIS

William N. Caballero, Capt, USAF

AFIT-ENS-MS-17-M-117

DEPARTMENT OF THE AIR FORCE
AIR UNIVERSITY

AIR FORCE INSTITUTE OF TECHNOLOGY

Wright-Patterson Air Force Base, Ohio

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THESIS

Presented to the Faculty

Department of Operational Sciences

Graduate School of Engineering and Management

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Air Education and Training Command

in Partial Fulfillment of the Requirements for the

Degree of Master of Science in Operations Research

William N. Caballero, B.S.I.E.

Capt, USAF

23 March 2017

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THESIS

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AFIT-ENS-MS-17-M-117

Abstract

Burden sharing within an international alliance is a contentious topic, especially in the current geopolitical environment, that in practice is generally imposed by a central authority's perception of its members' abilities to contribute. Instead, we propose a cost sharing mechanism such that burden shares are allocated to nations based on their honest declarations of the alliance's worth. Specifically, we develop a set of multiobjective nonlinear optimization problem formulations that respectively impose Bayesian Incentive Compatible (BIC), Strategyproof (SP), and Group Strategyproof (GSP) mechanisms based on probabilistic inspection efforts and deception penalties that are budget balanced and in the core. Any feasible solution to these problems corresponds to a single stage Bayesian stochastic game wherein a collectively honest declaration is a Bayes-Nash equilibrium, a Nash Equilibrium in dominant strategies, or a collusion resistant Nash equilibrium, respectively, but the optimal solution considers the alliance's central authority preferences. Each formulation is shown to be a nonconvex optimization problem. The solution quality and computational effort required for three heuristic algorithms as well as the BARON global solver are analyzed to determine the superlative solution methodology for each problem. The Pareto fronts associated with each multiobjective optimization problem are examined to determine the tradeoff between inspection frequency and penalty severity required to obtain truthfulness under stronger assumptions. Memory limitations are examined to ascertain the size of alliances for which the proposed methodology can be utilized. Finally, a full block design experiment considering the clustering of available alliance valuations and the member nations' probability distributions therein is executed on an intermediate-sized alliance motivated by the South American alliance UNASUR.

To my wife, I pray that I've made you feel as loved and supported as you have I.

To my sons, being your father makes me a better man. I love you all.

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William N. Caballero

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ON PROPORTIONATE AND TRUTHFUL INTERNATIONAL ALLIANCE CONTRIBUTIONS: AN ANALYSIS OF INCENTIVE COMPATIBLE COST SHARING MECHANISMS TO BURDEN SHARING

I. Introduction

During the 2016 presidential campaign, NATO saw the reemergence in the American collective consciousness of a contentious topic from which it has been plagued since its formation: burden sharing. While the relevance of the alliance has not always been similarly questioned, the debate pertaining to the allocation of burdens and responsibilities has remained a concern among member nations, as witnessed by decades of negative media coverage concerning failures to meet the 2% of GDP defense spending requirement. However, measuring the fairness of an alliance's burden sharing can be obfuscated by considering metrics individually. For example, contradictions emerge when evaluating a subset of NATO's eleven existing metrics to ascertain a member state's contribution. Denmark has recently failed to meet both the 2% of GDP defense expenditure and the 20% of GDP Research and Development requirements, but it outperforms many other allies in contributions to recent conflicts [North Atlantic Treaty Organization, 2016]. Likewise, the United States continually surpasses these specific requirements, but it receives a substantive discount in its monetary contribution to the NATO common budgets. The American economy accounts for over 40% of the alliance's total GDP sum, but the United States is only required to provided 22.14% of the common fund budgets. When contributions are considered based upon common fund budgets and the size of each nation's economy, Germany bears the largest proportional share [Mattelaer, 2016]. Thus, depending

on which metrics are utilized, differing conclusions can be drawn regarding which country is or is not assuming its proper share of the collective burden. To this end, we focus on the sharing of alliance requirements such that the allocations exhibit a form of fairness as defined using coalitional game theory, and that it is in the best interest of member nations to honestly and transparently reveal their perception of the alliance's value. We will utilize the game theoretic notion of the core, removing emphasis on individual metrics, and replacing it by focusing on whether the country has incentive to be in the alliance. Should such incentive exist, we seek for nations to honestly report the maximum they are willing to contribute based on their own valuation of the alliance, rather than meet a goal from the central authority which may underestimate or inflate a nation's burden share. Specifically, we aim to achieve these goals with smaller regional, international alliances rather than larger, global alliances.

Whereas, the difficulties in allocating contributions among alliance members is apparent, history indicates the growth in prominence of interstate partnerships positively correlates with improving regional stability. An example of this can be observed within the context of the European Union. Born from the 1951 Schuman Plan, a simple economic agreement between six European countries gradually transformed into the European Economic Community in 1958 and to the 28-member European Union (EU) in 1993. The scope of the EU today has expanded from its humble beginnings as an economic partnership to encompass a melange of policy areas from climate, environment, and health to external relations and security, justice, and migration [European Union, 2016]. Differing views exist on the effectiveness and worth of EU policy in the region, but there can be little doubt that the EU is the preeminent regional, interstate partnership in the world.

Many regions throughout the world have labored to mimic the efficacy of the Euro-

pean Union, albeit with varying levels of success. Since the dissolution of the Spanish colonial footprint in the Americas during the mid-19th century, Latin America has never experienced an epoch of lasting regional stability. The United States' southern neighbor, Mexico, can be viewed as an exemplar of this instability. After eleven years of revolution, Mexico gained its independence in 1821 and, in the decades that followed, suffered a variety of civil and international wars, including the Texas War of Independence and the Mexican American War. Between 1833 and 1855, Mexico experienced 36 changes in the presidency. The Second Franco-Mexican War in 1862 installed an Austrian Hapsburg prince, Maximilian I, as emperor. This short-lived empire ceded way to the forty-year reign of dictator General Porfirio Diaz, which ended due to the Mexican Revolution and culminated in the uninterrupted line of presidents from the Partido Revolucionario Institucional (PRI) until 2000 [Krauze, 1998]. This *Dictadura Perfecta* (Perfect Dictatorship), as coined by Mario Vargas Llosa, and the presidency's transfer to the Partido Accion Nacional (PAN), arguably helped set the conditions that enabled the brutal confrontations with narcoterrorist that the country faces today [Grillo, 2012].

The unfortunate truth is that Mexico's tumultuous history is not unique in the region. Similar historical accounts can be seen throughout Latin America. Chile and Uruguay, currently bulwarks of South American democracy, were only within the last fifty years able to rid themselves of the dictatorial regimes of Pinochet and Bordaberry. We could expand this list even further should we include Panama's Noriega or Cuba's Batista. However, this political instability is not isolated to the past. Current regimes, such as those in Venezuela and Cuba, illustrate that dictatorial forces can still thrive in the region. Moreover, many democracies in Latin America are weak, lack political support, and are corrupt. These political realities, exacerbated by other major social problems, can create the conditions in which violence and

instability multiply. Malva Salvatrucha (MS-13), Barrio 18, and similar gangs plague society and made El Salvador the per capita murder capital of the world in 2015 [The Economist, 2016a]. Mexican narcoterrorists have waged war against a government that has struggled to contain them. In fact, evidence suggests Mexican drug cartels have facilitated the acquisition and smuggling of weapons by Hezbollah operatives into the United States, threatening to convert regional instability into global instability [Bartell & Gray, 2012].

In an effort to unite the region, multiple Latin American interstate partnerships have been created. The Organization of American States (OAS) includes 35 American sovereign nations and lists among its objectives (1) strengthening the peace and security of the continent, (2) promoting and consolidating representative democracy, and (3) promoting through cooperative action the region's social, economic, and cultural development [Organization of American States, 1993]. The OAS traces its roots back to 1890 and claims to be the oldest regional institution in the world. However in the last twenty years a variety of competing organizations have emerged. Another organization known as Union of South American Nations (UNASUR), comprised of only twelve South American states, has very similar objectives and was formed in 2010. The most significant differences between the two are the primary sources of funding and the member nation composition [Union of South American Nations, 2010]. Likewise, the Community of Latin-American and Caribbean States (CELAC), formed in 2011, also mirrors OAS objectives and has a similar composition, but with the exclusion of the United States and Canada [Community of Latin American and Carribean States, 2011]. The Caribbean Community (CARICOM), an alliance composed of primarily Caribbean island nations such as Haiti and the Dominican Republic, lists among its ten objectives (1) to affirm the collective identity and facilitate social cohesion of the people of the community, and (2) to ensure that social and economic

justice and the principles of good governance are enshrined in law and embedded in practice [Carribean Community, 2014]. Article 4 of the Treaty of Basseterre lists overlapping objectives for the Organization of Eastern Caribbean States (OECS), for which membership is fundamentally a subset of CARICOM [Organization of Eastern Carribean States, 2014]. The lines of distinction and separation of purposes become even less distinct when other interstate partnerships such as the Latin American Integration Association (ALADI), the Andean Community (CAN), the Association of Caribbean States (ACS), the Central American Integration System (SICA), the Amazon Cooperation Treaty Organization (ACTO) and others are examined as a whole.

The emergence of a single Latin American regional alliance is hampered by differences arising from disparate economies, ethnic makeups, and historical tensions, much as the European continent was, albeit without the specter of two world wars to promote regional unity. However, the world is observing a global resurgence of nationalism. The Economist [2014] noted this worldwide surge by citing the election of Indian Prime Minister Narendra Modi, often referred to as a Hindu nationalist; public support for France's National Front; and a Scottish movement which nearly separated it from the United Kingdom. This global trend has not subsided through 2016, as evidenced by both the Brexit referendum, and the French elections. The recently inaugurated American president, Donald Trump, has effused a distinct approach to foreign policy unlike any president since the second world war, creating a general mood of uncertainty in the international community [The Economist, 2016b]. In such an environment, alliances of all types are likely to be questioned for relevance and their funding structure scrutinized.

In general, the identification and implementation of an effective mechanism to fund an interstate organization is an essential task. Two common motifs exist across many

alliances and international agreements. The first, typically practiced in humanitarian organizations, is the voluntary solicitation of funds. In such a mechanism the organization's funding is, in effect, tithed without any strict requirements. The United Nations Office for the Coordination of Humanitarian Affairs (OCHA) is one such example. OCHA provides humanitarian aid worldwide. Only 5% of OCHA funding comes from the UN regular fund. The remainder of the budget is provided by charitable contributions from governments, non-government agencies, and other private entities [United Nations Office for the Coordination of Humanitarian Affairs, 2016a]. Two monetary funds managed by OCHA, country-based pooled funds (CBPFs), and the United Nations Central Emergency Response Fund (CERF), comprise the remaining resources utilized to complete its mission. In CBPFs, donors allocate an amount for country-specific relief in coordination with OCHA. The CERF aggregates donor contributions - mainly from governments but also from foundations - that can be used for worldwide disaster relief [United Nations Office for the Coordination of Humanitarian Affairs, 2016b]. However, for alliances pertaining to military or trade with larger budgetary requirements, a charitable donation method is less likely to generate the requisite funds and, by design, is unable to enforce a fair burden sharing methodology.

The second form of funding, as utilized by the United Nations, the UNASUR, the OAS, and NATO with regard to common budgets, is to allocate quotas among countries based on their capacity to pay [Organization of American States, 2005; Union of South American Nations, 2015]. This general process typically involves the division of payments based upon some metric of each respective country's economic strength (e.g., GDP, GNP, NNP), followed by adjustments based on population and national debt factor. The calculated quotas generally are bounded between some maximum and minimum values, or adjustments are performed until such constraints

are met. This methodology can be viewed as an analog of the Shapley value in coalitional game theory, as it attempts to fairly share payments among member states. However, countries do not always pay their requisite quotas to the coalition, as observed in Venezuela's failure to pay its OAS quota for many years [Morello, 2016]. Moreover, Greece has recently shown that national economic figures are not necessarily sacrosanct and can be falsified [Papaconstantinoy & Sachindis, 2016]. Thus, the capacity-to-pay method may be difficult to enforce and its underlying principles are subject to manipulation. In fact, neither of the two discussed funding methodologies serve to adequately solicit a country's true valuation of the alliance, as there exists the ability to withhold funds or act deceitfully with little consequence.

In the current research, we explore mechanisms that systematically share the burden of an alliance to each member country according to their reported reserve prices (i.e., the maximum the country is willing to contribute to the alliance). While, in our context, an agent (i.e., a country) is actually a collection of people and the agent's valuation is generally itself a social choice function, we will assume this value has already been determined. In this way, a country is asked to reveal its reserve price valuation of the alliance, and the mechanisms are designed to motivate honest declarations. We assume the leadership of an alliance is independent and credible. Alliance policies are therefore unbiased and enforceable. Assuming perfect inspection for this initial research, we formulate three multiobjective nonlinear programs for which the decision variables represent inspection probabilities and deception penalties. Any feasible solution constitutes a budget balanced mechanism in the core, wherein a collective truthful revelation of reserve prices by all countries is a Bayes-Nash equilibrium, a Nash Equilibrium in dominant strategies, and a collusion resistant Nash equilibrium, respectively, for our three problem formulations. However, the optimal solution to any of the three problems induces truthfulness while optimizing with respect to the

alliance's central authority preferences. We also examine the respective problems' convexity, the ability of multiple commercial solvers to find an optimal solution, the size limitations of solvable instances for all three problem formulations, as well as the Bayesian Incentive Compatible mechanism on a intermediate size alliance motivated by UNASUR.

The results of this research provide an upper bound on the effort required to induce honesty in member nations under the assumptions of perfect inspection and enforceable penalties such that cost shares are not explicitly set by a central authority but are derived from the nation's own valuation of the alliance. Likewise, we present a new reformulation of strategyproof and groupstrategyproof mechanisms as an optimization problem. However, the multiobjective nonlinear programs are not specific to alliance burden sharing or even international relations. The optimization problems presented herein are applicable to any general cost sharing problem with perfect, probabilistic inspection effort and penalties based upon dishonest reporting of valuations.

II. Literature Review

This research is focused on the application of cost sharing mechanisms to funding endogenous to the international partnerships. We begin by reviewing the fundamental mathematical principles underlying cost sharing mechanisms: game theory, social choice theory, and mechanism design. The study of cost sharing mechanisms is a subset of mechanism design, a field itself which can be viewed as the intersection of traditional game theory and social choice theory. Therefore, an understanding of these three areas of study is critical to explain our results. Following a review of these mathematical principles, we examine recent literature conducted on cost sharing mechanisms and discuss the importance of both cross-monotonic mechanisms and group strategyproofness. Finally, we conclude with a review of burden sharing in international partnerships.

2.1 Game Theory

Rooted in utility theory's fundamental proof that a player's preferences for outcomes can be represented by a scalar, game theory studies the interaction of selfish agents to find useful solution concepts [Neumann *et al.*, 1944]. Specifically, game theory examines many different forms of interactions between players such as simultaneous actions (normal form) and sequential actions (extensive form) with varying levels of knowledge of past moves (perfect vs. imperfect information) and player utilities (incomplete information). Game theory can be partitioned as a discipline based upon whether the modeling focus is the player or the group. Noncooperative game theory describes the former whereas cooperative, or coalitional, game theory describes the latter.

We begin by examining noncooperative games. In a perfect information normal

form game, the most famous among all solution concepts is the Nash Equilibrium: a set of players' strategies from which no player benefits by unilaterally changing his own strategy [Nash *et al.*, 1950]. The strength of the Nash equilibrium as a solution concept is its existence in every game having a finite number of players and action profiles. Other well studied solution concepts include the minimax and the maximin strategies, wherein a player's action respectively minimizes his opponents maximum payoff or maximizes his own minimum payoff. In fact, the Minimax Theorem proves that, in two player zero-sum games, all three of these solution concepts coincide as the same strategy set [Neumann *et al.*, 1944]. Further solution concepts such as Evolutionary Stable Strategies and ε-Nash equilibriums have refined the Nash equilibrium to specific requirements in normal form games. An evolutionary stable strategy strengthens the Nash equilibrium by requiring that no player benefit by changing strategies and players are all strictly better off by keeping the specified strategy [Shoham & Leyton-Brown, 2008; Smith & Price, 1973]. In contrast, an ε-Nash equilibrium weakens this constraint and specifies a strategy profile such that no player can gain more than a very small ε-amount of utility by changing strategies [Shoham & Leyton-Brown, 2008].

Beyond the perfect information normal form games, there exists a genre of games known as extensive form, with either perfect or imperfect information. Extensive form games introduce a temporal element missing in the simultaneous play of normal form games. In extensive form games, players move sequentially and, depending on whether the game is one of perfect or imperfect information, have varying amounts of information about preceding moves. Perfect information games assume players have accurate knowledge of all previous decisions. In analyzing such games, the Subgame Perfect Nash Equilibrium (SPNE) is a fundamental tool [Selten, 1965]. The concept of an SPNE draws on the fact that an extensive form game can be represented as

a game tree starting with the first player's decision at the root node, branching at successive players' decisions until the last round of play results in utilities identified for each player at a leaf (terminal) node. An SPNE is a strategy profile wherein, if the game tree is cut at any level (forming a subgame), the corresponding actions in the strategy profile correspond to a Nash Equilibrium for the subgame. Imperfect information extensive form games, introduced by Kuhn in 1953, imply that a player does not possess knowledge of all previous moves taken; he is unaware of his location on the game tree [Kuhn, 1953]. For this reason, SPNEs are an inadequate solution concept for this scenario, as a set of subgames may be indistinguishable from one another. In fact, the naïve application of SPNEs rules out all possible strategies [Shoham & Leyton-Brown, 2008]. Thus, in studying imperfect information extensive form games with perfect recall, sequential equilibrium specifying a strategy and belief distribution have proven to be very effective tools [Kreps & Wilson, 1982].

Perhaps the set of games most pertinent to the current research is that of Bayesian games, also known as incomplete information games. In this context, a group of players has knowledge of the strategy space available to all players but does not know for certain each player's utility of any outcome (i.e., each player's type is unknown). Equilibrium is usually computed via the Bayes-Nash solution concept for such games [Harsanyi, 1967]. In a Bayes-Nash equilibrium no player's expected utility increases by changing his own strategy under his beliefs of other players' strategies. This equilibrium concept is utilized when expected utility is calculated as *ex-ante* or *ex-interim*, wherein a player has no knowledge of any other player's type, but does have knowledge of only his own type. Utility can also be calculated *ex-post* such that every players' type is public knowledge. The solution concept utilizing this form of utility expectation is called *ex-post* equilibrium and is similar to Bayes-Nash equilibrium except for its use of *ex-post* expectation [Shoham & Leyton-Brown, 2008]. *Ex-post*

equilibrium is a stronger condition than Bayes-Nash, but it does not have the same existence guarantees present under the *ex-ante* assumption.

Transitioning to coalitional game theory, the primary questions addressed are what subset of players will form a coalition, whether the coalition is stable, and how the coalition should divide its transferable utility among its members. The question of equitable division of the utility has been famously addressed via the Shapley value [Shapley, 1988]. The Shapley value captures the average marginal contribution of each agent and distributes the payoff accordingly [Shoham & Leyton-Brown, 2008]. While it answers the question of payment allocation, it does not provide any guarantee of coalition stability. That is, if the Shapley value is calculated and implemented for the grand coalition, there is no guarantee the coalition is stable. This stability question has been answered by a variety of concepts. The core of a coalitional game is the set of all payoffs such that no sub-coalition has an incentive to deviate from the current coalition [Gillies, n.d.]. It can therefore be viewed as an extension of Nash Equilibrium to the coalitional game setting. However, there is no guarantee that the core of a coalitional game is not empty. For this reason, the ε-core, where no subcoalition gains more than ε-amount of utility by deviating, and the least core, the smallest ε-valued ε-core that exists for a game, have been developed. If ε is too small, the ε-core may still be empty. However, the least core will never be empty as it comprises the set of vectors that solve the ε-value minimization problem [Shoham & Leyton-Brown, 2008].

2.2 Social Choice Theory

The theory of social choice concerns the aggregation of individual preferences in a society essentially from a coalitional game theoretic perspective. That is, social choice theory is concerned with the maximization of social welfare. However, it is as-

sumed all agents freely reveal their preferences truthfully and are therefore not selfish. Social choice theory models the action of a central authority seeking to accumulate the information provided by the agents in a logical and coherent manner to make a decision. The central authority will return either a single outcome via a social choice function or a ranking of all outcomes via a social welfare function developed from some voting scheme (e.g., plurality voting, cumulative voting, and Borda voting). An ideal social choice function is weakly Pareto efficient, monotonic, and nondictatorial [Shoham & Leyton-Brown, 2008]. That is, the social choice function will not select an outcome that is dominated by another outcome, its selection remains the same as more supporters of the choice are added, and no single voter completely determines the central authority's selection. Unfortunately, all three of these conditions cannot be met simultaneously. It has been proven that a Pareto efficient and monotonic social choice function must be dictatorial [Muller & Satterthwaite, 1977]. A similar result holds for social welfare functions in that they cannot be simultaneously Pareto efficient, independent of irrelevant alternatives (i.e., dependent only on the relative ordering given by agents), and non-dictatorial when there exists more than two alternatives [Arrow, 1970]. Both of these impossibilities are often addressed by relaxing the appropriate Pareto efficiency, independence of irrelevant alternatives, or monotonicity requirements. Moulin demonstrated two such approaches for social welfare functions by restricting preferences to be single peaked and by restricting the social welfare relation to be only acyclic [Moulin, 1994].

2.3 Mechanism Design

Mechanism Design can be viewed as the intersection of game theory and social choice theory. We are still concerned with a central authority making a decision using players' declared preferences, but now under the assumption that each player is mo-

tivated by self-interest. Players may hide their true preference values if they perceive doing so will improve their individual outcome that results from a central authority's decision. Mechanism design works in a Bayesian setting wherein there exists a finite set of players, a set of outcomes, a set of possible player types, and a utility function for each player. When this Bayesian setting is combined with the mechanism (i.e., a set of actions available to the agents, and a mapping of action profiles to outcomes) the result is a Bayesian game. The mechanism is designed to induce desirable traits in the solution(s) to the Bayesian games [Shoham & Leyton-Brown, 2008]. For instance, the central authority may wish that the Bayesian game implements a particular social choice function in dominant strategies, in a Bayes-Nash Equilibrium, or in an *ex-post* equilibrium. Alternatively, the central authority may wish that the mechanism induces all agents to take non-deceptive actions (truthfulness), maximizes the sum of all players welfare (efficiency), or makes neither a profit or a loss (budget balanced). A fundamental theorem in mechanism design, the revelation principle, relates the implementation of a social choice function to truthfulness. It states that, if there exists a mechanism implementing a social choice function, there exists a direct and truthful mechanism that does the same [Gibbard, 1973]. This theorem enables the action space postulated in a mechanism to be restricted to only direct actions (i.e. actions wherein the player only reports his preference). Any solution to a mechanism design problem can be converted into one in which agents always reveal their true preferences [Shoham & Leyton-Brown, 2008]. In terms of mechanism implementation, dominant strategies implementation is the strongest, but it has been proven that any mechanism implementing a surjective social choice function having at least three outcomes that is dominant strategy truthful is also dictatorial [Satterthwaite, 1975; Gibbard, 1973]. Similarly, no strategyproof mechanism (i.e., one in which truth-telling is a dominant strategy) can simultaneously be efficient and budget-balanced

[Green & Laffont, 1977].

In the setting of quasilinear preferences (transferable utility), the Vickery-Clarke-Groves (VCG) mechanism is well studied and has beneficial properties [Vickrey, 1961]. The VCG mechanism is a direct mechanism that is strategyproof and efficient. Efficiency is maximized by selecting the combination of reported valuations which maximizes the social welfare of the group. Truthfulness is then enforced through a (potentially positive or negative valued) payment to each agent that does not depend on his own valuation [Vickrey, 1961; Clarke, 1971; Groves, 1973]. Although the VCG mechanism has been proven to be the only directly implementable mechanism that is both truthful and efficient, it is also known to be susceptible to collusion (i.e., it is not group strategyproof) and expensive to implement [Green & Laffont, 1977; Shoham & Leyton-Brown, 2008].

Essentially, mechanism design problems are optimization problems wherein the parameters are privately held information. Vohra demonstrates how incentive compatibility, revenue maximization, the core, and efficiency can be formulated using linear programming techniques and network models [Vohra, 2011]. His work is our primary motivation for the application of traditional operations research techniques to mechanism design problems.

2.4 Cost Sharing Mechanisms

Mechanisms designed to share the price of a good or service among many players are called cost sharing mechanisms. Research is focused on developing mechanisms that prescribe desired behavior for rational agents. These cost sharing mechanisms implement an underlying cost sharing function and are typically constrained by some variation of budget balance, no positive transfer (bidder is not paid to not receive a service), individual rationality (a player does not receive negative utility from not

playing), and voluntary participation (players payment doesn't exceed his bid) restrictions. The Shapley value mechanism and the VCG have been applied successfully to such problems [Shoham & Leyton-Brown, 2008]. In both of these cost sharing mechanisms, the goal of strategyproofness is accomplished. No single agent is able to deviate unilaterally and increase his utility. However, individual strategyproof mechanisms are susceptible to collusion. It is possible for groups of agents to coordinate their actions in such a way that truthfulness is not a dominant strategy. Mechanisms that are resistant to collusion are called group strategyproof if no subset of players can gain utility by collusion without hurting a member of the same subset. A further consideration is the inclusion of the cost sharing mechanism in the core. Cost sharing mechanisms implementing a cross monotonic cost sharing function will exist in the core whereas, for combinatorial optimization games, the core is non-empty only if the problem is unimodular (i.e., the linear program relaxation of the integer program corresponds to an integer valued basic feasible solution) [Deng *et al.*, 1999].

The study of group strategyproof mechanisms was first accomplished by Moulin [Moulin, 1999; Moulin & Shenker, 2001]. It was shown that if the underlying cost sharing function is cross-monotonic and budget balanced (i.e., no player is made to pay more as the size of the coalition increases and the cost is met exactly), there exists a group strategyproof mechanism in the core to implement it. Therefore, the mechanism induces a stable cost sharing scheme wherein a subset of colluding players will not all benefit. In a subset of colluding players, at least one of them will be impacted negatively from this action. This notable result has been extended by many authors and, for a number of years, was the only known method for constructing group strategyproof mechanisms. While the underlying structure of group strategyproof budget balanced cost sharing mechanisms was explored and defined, it remained an open question as to whether there existed another method beyond Moulin mechanisms

[Penna & Ventre, 2006]. Moulin mechanisms, as they have come to be known, can be thought of as an iteratively ascending auction. In each iteration, players are simultaneously offered the same price. If a player refuses in a given iteration, the offered price is incremented and offered to the remaining players [Tazari, 2005; Mehta *et al.*, 2009]. The mechanism terminates when all players accept a given offer. Moulin mechanisms have been successfully applied to many situations, including cost sharing in the electronic market place [Li *et al.*, 2003]. However, in some situations cross monotonicity severely limits the effectiveness of a mechanism. Li et al. (2010) introduced a cross monotonic cost sharing scheme for the set covering problem, but it is not budget balanced and is limited to recovering at most $\frac{1}{2n}$ of the group cost [Li *et al.*, 2010a,b]. In fact, utilizing the results of Jain & Vazirani [2001] that show Moulin's theorem holds for α-budget balanced cost sharing schemes, it has been shown that cross monotonic cost sharing functions for the vertex cover and set cover problems can recover at most $\sqrt[3]{n}$ and $\frac{1}{n}$ of the group cost, respectively. In contrast, Moulin mechanisms for the facility location game were shown to recover at most $\frac{1}{3}$ of the group cost [Jain & Vazirani, 2001; Immorlica *et al.*, 2008].

The poor budget balance results motivated researchers to relax the core and group strategyproof attributes of Moulin mechanisms to an approximate level in an effort to increase the budget balance factor. Acyclic mechanisms represent one such effort. These mechanisms, similar to Moulin mechanisms, can be understood as an iteratively increasing auction, but they differ in the price offering method. Whereas Moulin mechanisms simultaneously offer a price share to all participants, acyclic mechanisms do so in sequence. Once a player refuses an offer, the iteration terminates and the price share increases. All players who have not yet rejected a proposal are then offered the new price in sequence. The mechanism continues in this way until all players accept the offered price. While it has been shown that acyclic mechanisms

generalize Moulin mechanisms when a null timing function is input (i.e., all players are offered a price share simultaneously), acyclic mechanisms generally have weaker guarantees with regard to group strategyproofness. Specifically, acyclic mechanisms are individually strategyproof but weakly group strategyproof. That is, a subset of colluding players cannot all benefit, as some of the colluding players will remain indifferent between selecting a truthful or a colluding strategy. Although this is a weaker guarantee than group strategyproofness, the combination of the non-decreasing nature of offered price shares and the timing function allows acyclic mechanisms to utilize cost sharing functions that are not cross-monotonic. Many primal-dual algorithms naturally induce acyclic mechanisms having better performance guarantees than Moulin mechanisms in terms of social welfare maximization and budget balance [Mehta *et al.*, 2009]. In fact, acyclic mechanisms for the set cover game and facility location game have been implemented and recovered $\frac{1}{\log n}$ and $\frac{1}{1.861}$ of the respective group cost [Devanur *et al.*, 2005]. Brenner & Schäfer [2009] have since demonstrated a method for turning any α-approximate algorithm into an α-approximate budget balanced mechanism.

Egalitarian mechanisms are a subclass of acyclic mechanisms. Egalitarian mechanisms find the most cost-efficient subset of players that have not been assigned a cost share and then charge the players in the set an identical minimum marginal cost. This process continues until all players have been assigned a cost. Egalitarian mechanisms are powerful as they possess a stronger notion of collusion protection than weak group strategyproofness. Egalitarian mechanisms have been proven to possess the property of weak group strategyproofness against collectors. This form of collusion protection strengthens weak group straegyproofness by the notion that all players prefer receiving service at their valuation price over not receiving it at all. It has been proven that the set of acyclic mechanisms also possesses the property of

weak group strategyproofness against collectors [Bleischwitz *et al.*, 2007].

Upon researching the effects of relaxing the requirements of Moulin mechanisms to improve budget balance and social welfare, it was discovered that group strategyproof mechanisms can be implemented with underlying cost functions that are not cross monotonic. Instead, group strategyproof mechanisms need only satisfy the weaker notion of fence monotonicity. Cross monotonicity refers to the properties of the underlying cost function, whereas fence monotonicity pertains only to the allocated price shares. A group strategyproof mechanism is completely characterized by satisfying fence monotonicity, combined with the stability of its allocations and the validity of a tie-breaking rule [Pountourakis & Vidali, 2010]. A fully budget balanced group strategyproof mechanism for the unweighted edge cover problem has been created based upon the concept of fence monotonicity [Immorlica & Pountourakis, 2012]. This finding represents a substantial improvement from the previously proven upper bound of $\frac{1}{2}$-budget balanced for cross monotonic group strategy proof mechanisms. In the same paper, an upper bound of $\frac{18}{19}$ budget balance is provided for the fence monotonic mechanism in the set cover problem, improving upon the $\frac{1}{n}$ upper bound for Moulin mechanisms.

The aforementioned cost sharing mechanisms approach the problem in a coalitional game theoretic manner. In such a scenario, there exists an implicit assumption that a central authority exists to facilitate the players' interaction. Decentralized environments for which no central governing body exists have been modeled utilizing techniques from non-cooperative game theory. In many situations, such as those often considered by computer science pertaining to the Internet, a centralized model for cost sharing is impossible to implement. Cardinal & Hoefer [2010] considered a vertex cover game wherein each player is responsible for a subset of edges on a graph. Each player insists upon all of his edges being adjacent to at least one node. In this

way, the players can be considered to be owners of related constraints in the vertex cover integer program formulation. The authors only considered pure strategy Nash equilibrium and analyzed the game primarily with regards to the price of anarchy and the price of stability, i.e., the ratio of the worst and best Nash equilibrium to the solution which maximizes social welfare. As pure strategy Nash equilibrium are not always guaranteed to exist, the authors utilize approximate (α, β)-Nash equilibrium and introduce algorithms for finding them in multiple classes of problems, including the use of primal dual algorithms. Other works have analyzed the effect of signaling or analyzed covering games with a low price of anarchy [Balcan *et al.*, 2014; Piliouras *et al.*, 2015]. However, one of the most significant works in the area exposed the relationship between non-cooperative cost sharing and coalitional cost sharing mechanisms. If a strong Nash equilibrium exists in the strategic (non-cooperative) form of the game, then the coalitional form of the game has a non-empty core [Hoefer, 2010]. While the reverse is not always true, the conditions under which strong equilibrium in cost sharing games exists has been codified [Epstein *et al.*, 2009].

2.5 Burden Sharing in International Affairs

Whereas cost sharing mechanisms adopt a prescriptive approach, the political science and economics literature generally adopt a descriptive approach to address the problem of burden sharing within nation-state alliances. Research on burden sharing in international affairs relates to the amount of absolute and relative contributions by players, typically countries, to the financing of a public good. Public goods are not divisible and are not excludable. Cost sharing mechanisms specific to public goods have been researched extensively in the literature. Jackson & Moulin [1992] consider the sharing of cost for an indivisible public project among many players, and their work was extended by Bag [1997] to account for a divisible project (im-

pure public good) and freeloaders. However, the most prevalent example of a public good is national defense, and it is to this end that the majority of burden sharing research literature is applied. NATO specifically has garnered much attention in this area. Olson & Zeckhauser [1966] outline a model explaining the contributions of countries to a coalition with specific emphasis on defense alliances. This model was then empirically compared with 1965 NATO funding levels to confirm the thesis of the exploitation of the great by the weak. That is, the model explains the disproportionate investment of large countries (in terms of population or GNP) to NATO. Kim & Hendry [1995] conduct a thorough survey of burden sharing research specific to defense alliances and find two major quantitative methods for approaching the problem: (1) economic/political analysis of alliance burden sharing, and (2) operations research analysis. While Kim and Hendry also address economic/political analysis of individual country's defense expenditures, the research in this area is much more qualitative. The following publications are exemplars of Kim and Hendry's aforementioned quantitative categories. Weber & Wiesmeth [1991] create an economic model of NATO, expanding upon Olson and Zeckhauser, applying a cost sharing mechanism to extract each player's payment, which is a quasi-egalitarian equilibrium and in the core. The mechanism is stable, efficient, and budget balanced. However, by the results of Green & Laffont [1977] it cannot be strategyproof. Their research also demonstrated how nonlinear defense expenditures can contradict the results of Olson & Zeckhauser [1966]. The operations research analysis of burden sharing is best exemplified by Hens *et al.* [1992] who apply a multi-criteria decision making model to identify each country's percentage burden share for the alliance. Their method can be viewed as an effort to generalize traditional practices currently imposed by the UN and the OAS. A set of representative criteria for burden sharing is selected, such as GNP and national debt as used by the UN, and percentage burden shares

are subsequently allocated by the model. Notably, all of the aforementioned methods model nations as individual actors, masking the effect of their respective populations. Boadway & Hayashi [1999] recommend an alternative approach which assumes that nations act to maximize the utility of their populations and confirms Olson and Zeckhauser's exploitation of the great by the weak hypothesis. Gupta *et al.* [2012] and Gupta [2014] utilize a sequenced voting scheme to invoke the efficient provisioning of international security under exogenous and endogenous threats.

The emphasis in burden sharing on defense alliances, and NATO in particular, is likely due to the global security environment at the time of each literature's publication, primarily during the Cold War. However, as noted by Chalmers [2001], the dominance of the military dimension in burden sharing is subsiding. Other public goods, such as financing EU enlargement, foreign aid to third world countries, and climate change, are beginning to gain preeminence. This reality is beginning to become apparent in the burden sharing literature as well. Böhringer *et al.* [2015] have produced one of the first examples analyzing the effect on the Canadian economy of six different mechanisms sharing the carbon emissions reduction burden among provinces per the Copenhagen Accord. However, many research opportunities remain pertaining to burden sharing within non-defense related alliances.

III. Methodology

In this section, we describe the proposed cost sharing mechanisms, introduce requisite terminology and parameters, and identify multi-objective nonlinear programming optimization formulations that identifies mechanism thresholds yielding various guarantees of truthfulness. Specifically, we introduce constraint sets that can be utilized by the mechanism designer to implement the cost sharing function in a Bayes-Nash equilibrium, and in dominant strategies with or without collusion resistance, respectively. The intent is to provide a framework with which a central authority can select inspection and penalty thresholds to deter deceptive actions.

3.1 Mechanism Intuition

Funding methods utilized by international organizations generally do not assess a country's value of the alliance directly. Instead, some sort of equitable price share is often developed based on a capability-to-pay method which utilizes national economic indicators and national debt as input variables. This method ignores any strategic importance the country may place on being in the alliance. Therefore, we propose a direct mechanism that asks each nation to reveal their valuation of the coalition. A cost sharing function then utilizes all member nations' reserve prices to determine their respective cost shares. We will define the valuation, or type, of each nation by their reserve price in terms of the percentage of the total alliance cost. We assume each country's valuation of the alliance is computed rationally using the principles of decision theory to balance internal and external risks/rewards (i.e., domestic political environment or aggressive responses by nonmember states).

Should the mechanism be left as such (i.e., each nation is only asked to reveal their reserve price), a country is generally incentivized to underreport their true valuation

if it believes other member states will sufficiently fund the endeavor. To deter such deception, the mechanism requires an enforcement protocol. The proposed enforcement protocol is a probabilistic inspection action by the central authority combined with a penalty imposed on any country found, via inspection, to be untruthful. Each bid in a countable valuation space will be probabilistically inspected independent of other valuations. For this study, inspection by the central authority is assumed to be perfect and without cost. That is, if the central authority decides to inspect a country, it is guaranteed to identify the nation's true type and the inspection action has neither a fixed nor a variable cost. The mechanism will allow truthful countries to be eligible for a subsidy and deceptive countries to be vulnerable of a penalty. If a country is found to be deceptive, the bid will be corrected to the true value and a penalty will be assessed. Countries that have not been labeled as deceptive will be rewarded, as the sum of deception penalties assessed will be equitably distributed as a subsidy among them.

Under the premise of perfect inspection, it becomes obvious that a probability one of inspection and/or a sufficiently large penalty for deceptiveness will incentivize truthfulness. Such an authoritarian enforcement protocol may be damaging to the alliance and difficult to enforce. Therefore, we desire to minimize inspection probabilities and the magnitudes of deception penalties while maintaining some form of truthfulness and budget balance. Likewise, the stability of the grand coalition is paramount. If the cost sharing mechanism induces instability, the international alliance itself is at risk of dissolution. We address such concerns utilizing a cross-monotonic cost sharing function to ensure a core allocation. However, the ensuing cost sharing mechanism can also allow for the alliance to go unfunded, in effect dissolving, if after inspection the sum of updated reserve prices is less than the total cost of the alliance. In such a scenario, the assessment is that the alliance has not

garnered sufficient support to continue. However, to maintain truthfulness, penalities and subsidies would still be assessed in such a situation.

The central authority provides a set of declarable reserve prices and a set of distributions for each country over them. This fact, combined with the stochastic inspection, signifies the mechanism induced game is a single stage Bayesian stochastic game. All of the ensuing mechanisms will utilize the expected cost share and penalty sum from a declaration as the measure of an action's utility. However, the three mechanisms differ in how the Bayesian nature of the game is managed.

3.2 Bayesian Incentive Compatible Formulation

We begin the discussion of our multi-objective nonlinear optimization problem formulations by introducing the requisite sets, parameters, and decision variables in our international alliance setting.

Sets and Parameters

- $\Phi = \{\phi_1, \phi_2, ..., \phi_n\}$: the set of all n possible types (valuations) in a common pool available to be the declared or truthful type of a nation.
- $\mathcal{M}$: the set of m countries in the alliance.
- $\mathcal{C}$: a set of colluding countries such that $\mathcal{C} \subseteq \mathcal{M}$.
- $\theta = \{\theta_1, \theta_2, ..., \theta_m\}$: the *true* type vector of the m players, wherein $\theta_j \in \Phi$, for j = 1,2,...,m, and where $\Theta = \Phi^m$ is the set of all possible *true* type vectors of the nations in the alliance.
- $S = \{s_1, s_2, ..., s_m\}$: the *declared* type vector of the m players in the alliance, wherein $s_j \in \Phi$, for j = 1,2,...,m, and where $\mathcal{S} = \Phi^m$ is the set of all possible *declared* type vectors of the nations in the alliance.

- $W = \{w_1,...,w_t\}$: the vector of inspected types such that $W \subseteq \Phi$
- $S^W = \{s_1^W,...,s_m^W\}$: the vector of *corrected* player types of S after inspections of declarations in W, wherein $s_j^W \in \Phi$, for j=1,2,...,m.
- $\alpha_{\theta_i,j}$: the probability that the jth player is truly type θ_i.
- $\beta_{\theta_{-j}}$: the probability of $\theta_{-j} \subset \theta$, i.e., the probability all players other than player j have a true type, which we represent as θ_{-j} and calculate via

$$\beta_{\theta_{-j}} = \prod_{\substack{k=1,2,...,n \\ k \neq j}} \alpha_{\theta_k,k}.$$

- τ: number of countries labeled as truth telling after inspection.
- σ_L: number of countries found underbidding after inspection.
- σ_H: number of countries found overbidding after inspection.
- λ_j: a deception coefficient such that $\lambda > 1$ indicates discomfort to deception, $\lambda < 1$ indicates comfort and $\lambda = 1$ indicates ambivalence.
- $c^W(s_j \mid S_{-j})$: cost function yielding the cost share of the jth player playing s_j and all other players declaring S_{-j}, after an inspection W has modified S to S^W. The base model will assume a proportional rule such that

$$c^W(s_j \mid S_{-j}) = \begin{cases} \frac{s_j^W}{\sum_{k=1}^{m} s_k^W}, & \text{if } \sum_{k=1}^{m} s_k^W \geq 1 \\ 0, & \text{otherwise.} \end{cases} \tag{1}$$

Decision Variables

- $p = \{p_{\phi_1}, p_{\phi_2}, ..., p_{\phi_n}\}$: the inspection probability vector, wherein, p_{ϕ_i} denotes the probability that a bid of type ϕ_i is inspected.
- x_L: The penalty assessed to a deceptive action if $s_j < \theta_j$.

- x_H: The penalty assessed to a deceptive action if $s_j > \theta_j$.

We observe that all players' valuations are drawn from a common pool. Individual pools are possible and would require only minor changes to the model, but a common pool makes the most sense for the context of our problem. Also, note that Θ and $\mathcal{S}$ are identical sets representing all possible combinations of size m from Φ where types can be repeated. A nation's valuation of the alliance will be represented as their reserve price in terms of the maximum percentage of the alliance the country is willing to contribute. We assume that a country's valuation will not exceed the cost of the alliance, and thus, the set Φ will contain n discrete elements generally contained within the interval [0,1]. In some context, it may also make sense to narrow this interval via an alternative upper bound, B_U, and lower bound, B_L, such that $0 \leq B_L < B_U \leq 1$. A scenario with such bounds coincides with the UN or NATO setting where participating countries are established by treaty, and bounds on minimum and maximum contributions have historically been mandated.

The distinction between S^W and S is necessary to adjust price shares after the central authority has discovered deceitful action. Upon discovering deceit, the central authority will adjust all players' cost shares as appropriate to the updated type vector and assess penalties and subsidies as required. Furthermore, S^W will always be as truthful or more truthful than S (i.e, S^W will have greater than or equal to the number of truthful players in S). Note that $S^{\emptyset}$ is equal to S because no inspections have occurred.

We assume that the true type of each country is independent of the true type of any other country. Therefore, β_θ equals the product of the appropriate $\alpha_{\theta_i,j}$ variables. We note that this simplification may not always reflect reality. Neighboring countries, countries with strong economic ties, or countries with similar geopolitical interest may very well exhibit correlated behavior in terms of their valuation of the alliance.

However, the proposed model is flexible enough that such correlation effects can be incorporated with minor modifications.

The cost function $c^W(s_j \mid S_{-j})$ represents the manner in which the price is shared after bids have been adjusted per the inspection vector W. If the sum of corrected values meets or exceeds the alliance cost, the alliance cost shares are proportionally allocated as described in the top term of equation (1). Otherwise, if the corrected values do not suffice, the alliance is not funded. It can be observed that, in our proportional cost function, an overbid relative to a player's true type will result in a higher expected payment. Therefore, the additional x_H penalty will be driven down to zero in accordance with the objective functions to be introduced in equations (5) and (6). However, the following formulations will retain x_H as a decision variable to account for cost sharing functions wherein an overbid may result in a lower expected payment (e.g., Li *et al.* [2003]).

We now introduce the functions that will be utilized in a plurality of the constraints for the mathematical programming formulation to ensure a pure strategy Bayes Nash equilibrium exists in the game.

Functions

- $\chi_j^W(S, \theta)$: the penalty and subsidy function associated with the inspection W, a declared type vector S, and a true type vector θ.

$$\chi_j^W(S,\theta) = \begin{cases} x_L, & \text{if } s_j < \theta_j \text{ and } s_j \in W \\ x_H, & \text{if } s_j > \theta_j \text{ and } s_j \in W \\ -(\sigma_L * x_L + \sigma_H * x_H)/\tau, & \text{if } s_j = \theta_j \text{ or } s_j \notin W \end{cases} \tag{2}$$

where

$$\sigma_L = \sum_{k=1}^{m} I_k^L$$

$$\sigma_H = \sum_{k=1}^{m} I_k^H$$

$$\tau = m - \sigma_L - \sigma_H$$

and

$$I_j^L = \begin{cases} 1, & \text{if } s_j < \theta_j \text{ and } s_j \in W \\ 0, & \text{otherwise} \end{cases}$$

$$I_j^H = \begin{cases} 1, & \text{if } s_j > \theta_j \text{ and } s_j \in W \\ 0, & \text{otherwise.} \end{cases}$$

- $a(s_j, p, x_L, x_H \mid S_{-j}, \theta)$: the expected allocation function computes the expected cost share of the jth player assuming he plays s_j, given the other players declared types and the true type vector. The function has the following form:

$$a(s_j, p, x_L, x_H \mid S_{-j}, \theta) = \sum_{W \subseteq \Phi} [c^W(s_j \mid S_{-j}) + \chi_j^W(S, \theta)] \prod_{\substack{i=1,2,\ldots,n \\ \phi_i \notin W}} (1 - p_{\phi_i}) \prod_{\substack{i=1,2,\ldots,n \\ \phi_i \in W}} p_{\phi_i}. \quad (3)$$

To increase or decrease a country's cost share after the adjustments of the inspection action, the penalty and subsidy function, $\chi_j^W(S, \theta)$ is defined. If a country is found to be deceptive by underbidding (i.e., the nation's declared valuation is inspected and its valuation is less than its true valuation), a penalty of x_L is assessed. Under parallel conditions, wherein the declared valuation is greater than the true valuation, the country is said to have overbid and is assessed a penalty of x_H. The

number of underbidding and overbidding countries are discerned through the sum of the indicator variables, I_j^L or I_j^H, and are represented by σ_L and σ_H, respectively. The number of countries who are deemed to be truth telling is represented by τ. Truth telling countries are either assessed and found to be honest or are not inspected at all and assumed to be honest. In either scenario, truth telling nations are eligible for a subsidy based upon the amount of penalties assessed to deceitful nations. This subsidy is shared equally among all truth telling nations.

The expected allocation function sums over all possible outcomes of the random variable W, ranging anywhere from no types are inspected to all types are inspected. For an individual outcome of W, the nation will be assigned a cost share and an appropriate penalty or subsidy. Within the summation over $W \subseteq \Phi$ in the right hand side of equation (3), each term consists of the sum of the cost share and the subsidy function, given a particular true type vector, declared type vector, high penalty, low penalty, and inspection vector W, multiplied by the probability of an inspection vector W occurring. Thus, the expected allocation function assesses the expected burden share of player j declaring s_j. For notational simplicity, we henceforth refer to this function as $a(s_j \mid S_{-j}, \theta)$ because p, x_L, and x_H are decision variables within the ensuing math programming formulation.

Given this requisite modeling framework, we introduce the nonlinear multiobjective optimization problem formulations associated with an international alliance cost sharing scenario. The first formulation seeks to balance the minimization of functions $f(p)$ and $h(x_H, x_L)$, whose specific forms are discussed later, while maintaining Bayesian Incentive Compatibility (BIC) internal to the mechanism. To induce a Bayes-Nash equilibrium, it must be assumed that the players in the partially filled declared strategy vector, S_{-j}, are reporting their true valuation. It is also important to note than any feasible solution to the optimization problem is a BIC budget

balanced mechanism, resulting in a game with a Bayes Nash equilibrium of honest strategies for all players. However, we wish to find an optimal mechanism with regard to minimizing the inspection probabilities and the magnitudes of the deception penalties.

BIC Formulation

$$\min_{p,x_H,x_L} \; z = (f(p), h(x_H, x_L)) \tag{4a}$$

subject to

$$\begin{aligned} &\sum_{\theta\in\Theta} \beta_{\theta_{-j}} a(\theta_j \mid S_{-j}, \theta) \\ &\qquad \leq \sum_{\theta\in\Theta} \lambda_j \beta_{\theta_{-j}} a(s_j \mid S_{-j}, \theta), \quad \forall j \in \mathcal{M}, s_j \neq \theta_j, \theta_j \in \Phi, \end{aligned} \tag{4b}$$

$$0 \leq p_{\phi_i} \leq 1, \quad \forall \phi_i \in \Phi, \tag{4c}$$

$$x_L, x_H \geq 0. \tag{4d}$$

The BIC formulation is multiobjective and can be approached utilizing any one of the available methods in the literature (e.g., see Caramia & Dell'Olmo [2008]). It can be observed that the first constraint set (4b) in the formulation enforces Bayesian Incentive Compatibility, assuming *ex-interim* utility (i.e., a country is aware of its own type, but not of other countries' types). While it would be possible to approach the problem under *ex-ante* assumptions (i.e., a country has no knowledge of any country's type), the definition of deception would become opaque if a country is not aware of its own type. Constraint set (4b) ensures the expectation of the expected cost share of declaring a true valuation will be less than or equal to the expectation when misrepresenting, for each nation and true type. Feasible solutions ensure a collectively honest declaration of worth by all nations is a Bayes Nash equilibrium. It is also this same truthfulness constraint which induces a nonlinearity. This is readily

noted when observing the product of multiple decision variables in equation (3). The second constraint set (4c) ensures that all inspection probability values are valid, and the last constraint set (4d) enforces non-negativity of the penalties.

We now introduce two potential forms of the objective function utilizing the weighted multi-objective method.

Individual Penalty and Inspection Weights (IPIW)

In lieu of equation (4a), we define the objective function to be

$$z = w_L x_L + w_H x_H + \sum_{i=1}^{n} w_i p_{\phi_i}, \tag{5}$$

wherein w_L and w_H weight the penalties x_L and x_H, respectively, and w_i-parameters weight the probability in which type ϕ_i is inspected, for $i = 1, ..., n$.

Penalty Sum and Inspection Effort (PSIE)

We replace equation (4a) with the objective function

$$z = w_P(x_L + x_H) + w_I \left(\sum_{i=1}^{n} p_{\phi_i} \right), \tag{6}$$

wherein w_P weights the sum of penalties and w_I weights the sum of type inspection probabilities.

Both of these forms are advantageous in some respects and limiting in others. When utilizing the IPIW objective function, a high degree of specification of preferences is possible. For instance, an alliance may not wish to severely punish countries who overbid their true valuation and may accordingly place a large weight on w_H. Likewise, if we remove the assumption of cost-free inspection, this method may be relevant and useful, especially if some bids require more inspection cost than others. The tradeoff to the robust specification capability is the difficulty in visualization

of the Pareto frontier in higher dimensions. The PSIE does not allow for the same specification as the IPIW objective function, but it allows for easy visualization of the Pareto frontier without regard to the size of the pool of potential valuations, and so we use it herein to illustrate results.

The solution of the optimization problem will yield the inputs for the central authority's enforcement protocol, i.e., the penalties for deceit and the probability of inspection for each valuation will be specified. Utilizing these inputs and the aforementioned method of soliciting inputs with the enforcement protocol, the central authority will have a budget balanced, Bayesian Incentive Compatible mechanism in the core at their disposal. However, should the above methodology be altered such that the cost function is no longer cross monotonic, there is no guarantee the resulting cost share allocation will be in the core.

3.3 Alternative Formulations

Our two proposed alternative formulations differ only in the type of incentive compatibility induced by the mechanism. That is, the solution to the optimization problem will induce penalties and probabilities such that either strategyproofness or group strategyproofness is induced.

Strategyproof Constraints (SP)

The strategyproof formulation replaces the first constraint set in the BIC formulation inequality (4b) with the constraint set (7), to induce a game wherein every player's weakly dominant strategy is to act honestly.

$$a(\theta_j \mid S_{-j}, \theta) \leq \lambda_j a(s_j \mid S_{-j}, \theta), \qquad \forall j \in \mathcal{M}, S_{-j} \cup s_j \in \mathcal{S} : s_j \neq \theta_j, \theta \in \Theta \tag{7}$$

Group Strategyproof Constraints (GSP)

The group strategyproof formulation replaces the first constraint set in the BIC formulation inequality (4b) with a similar set of constraints as in the strategyproof formulation, but constrains the expected contribution of groups to ensure no player can subsidize another to be dishonest and all players benefit. For the following GSP constraints, we must first define

- $\hat{S} = \bigcup_{j \in C} s_j$: the declared strategy vector of all colluding players in $\mathcal{C}$.
- $\hat{\theta} = \bigcup_{j \in C} \hat{\theta_j}$: the true type vector of all colluding players in $\mathcal{C}$.
- $\hat{S'} = \bigcup_{j \notin C} s_j$: the declared strategy vector of players not in the colluding set.

The following constraint set (8), when substituted in the BIC formulation for inequalities (4b), induces a weakly groupstrategyproof mechanism:

$$\sum_{j \in \mathcal{C}} a(\theta_j \mid \hat{\theta}_{-j} \cup \hat{S'}, \theta) \leq \sum_{j \in \mathcal{C}} \lambda_j a(s_j \mid \ \hat{S}_{-j} \cup \hat{S'}, \theta), \qquad \forall \mathcal{C} \subseteq \mathcal{M}, \hat{S} \cup \hat{S'} \in \mathcal{S} : \hat{S} \neq \hat{\theta}, \theta \in \Theta. \quad (8)$$

It can be observed that, in the strategyproof variant of the truthfulness constraints, we are no longer concerned with the expectation of the expected cost share of player j telling the truth. Instead, each expected cost share $a(s_j \mid S_{-j}, \theta)$ is considered individually. We now specify that regardless of the true type of declared valuations of the other players, player j's best response is to always tell the truth. This indicates that for all potential games in the Bayesian setting there exists a Nash equilibrium in weakly dominant strategies that is truth telling for all nations (adding a small scalar ε to the right hand side of the inequality induces an equilibrium in strictly dominant strategies). The group strategyproof variant of truthfulness expands upon the strategyproof definition by allowing for and preventing collusion.

In the previous settings, we have assumed that no communication between agents occurred to enable collusion. In the group strategyproof setting, it can be viewed as

if we are allowing any subset of agents to freely communicate with each other in an effort to coordinate their bids. The constraints in this setting will yield solutions to the optimization problem such that the penalty and probability values associated with deceitful action are a sufficiently large deterrent that, even with collusion, any player's best response remains to tell the truth for all potential games. It can be observed that when C is a singleton set (i.e., a player is considering unilateral deception), constraint (8) is identical to constraint (7) and will prohibit unilateral deception. Thus, the SP constraints and contained in the GSP constraints. Consider the following example to illustrate the difference between the SP and GSP formulations. Under joint truthful reporting Player A and B incur a cost of 0.2, and under joint deception Player A incurs a cost of 0 while Player B incurs a cost of 0.3. Such an outcome would not violate (7) as the joint truthful and joint deceitful strategies are never compared. Thus, Player A and B could benefit from collusion by splitting their joint cost (e.g., Player A and B both pay 0.15). Instead constraint (8) ensures the cost incurred by any colluding subcoalition acting truthfully is less than or equal to acting deceitfully, thereby preventing a player to benefit by subsidizing another. As in the SP constraint set, adding a small scalar ε to the right hand side of the inequality induces a group strategyproof instead of a weakly group strategyproof cost sharing mechanism.

The outlined frameworks allow for an international alliance to solicit and obtain preference information from its member nations and affect a selected expectation of truthfulness (e.g., BIC, strategyproofness, or group strategyproofness), assuming all actors are rational. The SP and GSP formulations no longer directly depend on the probabilities, $\alpha_{\theta_i,j}$, but this underlying information can still be utilized by the players to inform which game in the Bayesian setting is being played.

In some instances, the central authority may wish to avoid undesirable outcomes from irrational agents. While the mechanisms resulting from any of the aforemen-

tioned formulations will induce a budget balanced outcome in equilibrium, irrational actors may cause outcomes that overcollect, pay a country to be in the alliance, or assess penalties when the alliance is unfunded. Overcollection may occur when all countries act dishonestly and the alliance is funded. If all countries are found to be deceptive, no country is eligible for the subsidy and the money must either remain with the central authority or be disposed of similar to the VCG mechanism. In the same situation but when a small number of countries are deemed to be honest, said countries may be positively compensated for their participation (i.e., they are paid to be in the alliance). Finally, if declared bids do not meet the collective requirement, the alliance may remain unfunded, and our mechanism will still inspect and assess penalties to countries. Thereby, there exists the opportunity for honest countries to be compensated by deceptive countries while the alliance remains unfunded. To account for the last two scenarios, we introduce a *no positive transfer* constraint (i.e., no player is paid for participating).

No Positive Transfer Constraint

$$a(s_j \mid S_{-j}, \theta) \geq 0 \qquad \forall j \in \mathcal{M}, \theta \in \Theta, S_{-j} \cup s_j \in \mathcal{S},$$

This constraint set will prevent irrational actors from forcing the aforementioned undesirable outcomes. However, its inclusion into the BIC, SP, or GSP formulations may cause an instance to be infeasible.

3.4 Convexity of Formulations using BIC, SP, or GSP Constraints

The convexity of our non-linear formulations will determine the type of solution methodologies at our disposal. Given our objective function is linear, should the feasible region be convex, then convex optimization methods such as gradient-based procedures can be utilized to guarantee optimality. Otherwise, a global solver or

heuristic procedure will be required. In order for a feasible region to be convex, the constraints when expressed in standard form (i.e., $g_i(\vec{x}) \leq 0, i = 1, ..., m$) must also be convex. The literature demonstrates two popular methods to determine the convexity of a function. The first method as discussed by Winston & Goldberg [2004] utilizes the principal minors of the Hessian. The signs of the determinants of the principal minors determine whether or not the Hessian is positive definite or semidefinite, the former implying strict convexity of the function and the latter convexity of the function. Similarly, Bazaraa *et al.* [2013] utilize an equivalent approach based on the eigenvalues of the Hessian; a Hessian having all non-negative-valued eigenvalues corresponds to a convex functions, whereas a Hessian having all positive-valued eigenvalues corresponds to a strictly convex function.

Theorem 3.4.1 *The BIC formulation is a non-convex optimization problem.*

Proof By contradiction, assume the BIC formulation induces a convex optimization problem. Therefore, all of the functions comprising the BIC constraint set are convex, such that

$$g_j^{\theta_j, s_j}(p, x_L, x_H) \leq 0, \quad \forall j \in \mathcal{M}, \theta_j \in \Phi, s_j \neq \theta_j,$$

where

$$g_j^{\theta_j, s_j}(p, x_L, x_H) = \sum_{\theta \in \Theta : s_j = \theta_j} a(\theta_j \mid S_{-j}, \theta) \beta_{\theta_{-j}} - \lambda_j \sum_{\theta \in \Theta : s_j = s_j} a(s_j \mid S_{-j}, \theta) \beta_{\theta_{-j}}. \tag{9}$$

Therefore, the Hessian, $H_j^{\theta_j, s_j}(p, x_L, x_H)$ of $g_j^{\theta_j, s_j}(p, x_L, x_H)$ is positive semidefinite (i.e., having strictly non-negative eigenvalues) for any player j.

Consider a problem instance having $\Phi = \{0.5, 0.7\}$ and $\mathcal{M} = \{\text{A,B,C}\}$, $\lambda_j = 1$ for all players, and $\alpha_{\theta_i, j}$ identical for every player strategy pair. In this setting, for Player

A, setting $\theta_A = \{0.7\}$ and $s_A = \{0.5\}$, the function $g_A^{0.7,0.5}(p_{0.5}, p_{0.7}, x_L, x_H)$ reduces to $g_A^{0.7,0.5}(p_{0.5}, p_{0.7}, x_L, x_H) = \left(\frac{96}{323}\right) - 4p_{0.5}x_L - \left(\frac{96}{323}\right) p_{0.5}$, and $H_A^{0.7,0.5}(p_{0.5}, p_{0.7}, x_L, x_H)$ has the following form:

$$H_A^{0.7,0.5}(p_{0.5}, p_{0.7}, x_L, x_H) = \begin{vmatrix} 0 & 0 & 0 & -4 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ -4 & 0 & 0 & 0 \end{vmatrix}.$$

The eigenvalues of $H_A^{0.7,0.5}(p_{0.5}, p_{0.7}, x_L, x_H)$ are $(-4, 0, 0, 4)$, which indicates that $H_A^{0.7,0.5}(p_{0.5}, p_{0.7}, x_L, x_H)$ is not positive semi-definite and $g_A^{0.7,0.5}(p_{0.5}, p_{0.7}, x_L, x_H)$ is not convex, resulting in a contradiction. ∎

Theorem 3.4.2 *The SP formulation is a non-convex optimization problem.*

Proof As with the BIC formulation, we assume that the SP formulation induces a convex optimization problem. Then all of the SP constraints are convex, such that

$$q_j^{\theta,S}(p, x_L, x_H) \leq 0, \quad \forall j \in \mathcal{M}, \theta \in \Theta, S_{-j} \in \mathcal{S}, s_j \neq \theta_j$$

where

$$q_j^{\theta,S}(p, x_L, x_H) = a(\theta_j \mid S_{-j}, \theta) - \lambda_j a(s_j \mid S_{-j}, \theta). \tag{10}$$

Consider a problem instance with $\Phi = \{0.5,\ 0.7\}$, $\mathcal{M}$= {A,B,C}, and λ_j =1 for all players. Again examining Player A but now with $\theta = \{0.7, 0.7, 0.7\}$ and $S = \{0.5, 0.5, 0.7\}$ (i.e., Player A and B are underbidding), we see that $q_A^{\theta,S}(p_{0.5}, p_{0.7}, x_L, x_H)$ reduces to $q_A^{\theta,S}(p_{0.5}, p_{0.7}, x_L, x_H) = \left(\frac{24}{323}\right) - \left(\frac{24}{323}\right) p_{0.5} - \left(\frac{3}{2}\right) p_{0.5}x_L$. The Hessian, $H_A^{\theta,S}(p_{0.5}, p_{0.7}, x_L, x_H)$ of $q_A^{\theta,S}(p_{0.5}, p_{0.7}, x_L, x_H)$ is equal to

$$H_A^{\theta,S}(p_{0.5}, p_{0.7}, x_L, x_H) = \begin{vmatrix} 0 & 0 & 0 & -\frac{3}{2} \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ -\frac{3}{2} & 0 & 0 & 0 \end{vmatrix}.$$

Its eigenvalues are $\{-\frac{3}{2}, 0, 0, \frac{3}{2}\}$, indicating that $H_A^{\theta,S}(p_{0.5}, p_{0.7}, x_L, x_H)$ is not positive semi-definite and $q_A^{\theta,S}(p_{0.5}, p_{0.7}, x_L, x_H)$ is not convex, resulting in a contradiction. ∎

Corollary 3.4.3 *The GSP formulation is a non-convex optimization problem.*

Proof The SP constraints are a subset of the GSP constraints. ∎

3.5 Relative Formulation Size Induced via BIC, SP, or GSP Constraints

The number of constraints increases as the truthfulness requirements strengthen. This result can be observed by inspecting the BIC, SP, and GSP constraint sets. For instances having a large magnitude of available strategies in Φ or having a large number of countries in $\mathcal{M}$, the generation of the constraint sets may become computationally burdensome. A similar effect is possible on the computational effort required to solve the problem. Therefore, in this section we investigate how the size of each constraint set relates to m and $\|\Phi\|$.

Assume there are three countries in an alliance and there are two available types for each country such that $\Phi = \{0.5, 0.7\}$. If each country has a positive probability of having a true type of any element in Φ, we have $\|\Theta\| = 8$ (i.e., the quantity of available true type vectors), as presented in Table 1. Given that player types are independent of each other and come from a common pool, these results can be generalized in the following manner:

$$\|\Theta\| = \|\Phi\|^m = \|\mathcal{S}\|. \tag{11}$$

Table 1. Formulation Size Example: All Type Vectors Existing in Θ by Country

A	B	C
0.5	0.5	0.5
0.5	0.5	0.7
0.5	0.7	0.5
0.5	0.7	0.7
0.7	0.5	0.5
0.7	0.5	0.7
0.7	0.7	0.5
0.7	0.7	0.7

From equation (11), it can be observed that the number of countries in the alliance has an exponential effect on the number of true strategy vectors available, whereas the cardinality of Φ has a polynomial effect for a given number of countries. Although $\|\Theta\|$ is most relevant to the size of the SP and GSP constraint sets, we will first utilize the aforementioned results to illustrate the size of the BIC constraint set.

Table 2. Formulation Size Example: $\alpha_{\theta_i,j}$ for Each Country and Type Combination

Type	A	B	C
0.5	0.5	0.1	0.5
0.7	0.5	0.9	0.5

The BIC formulation requires a probability distribution over Φ for each player. For our sample instance, these probabilities can be found in Table 2. Given this information, we will investigate the BIC constraint, given that Player A has $\theta_A = 0.7$. We know that the possible combinations of reported types is the same as the possible number of true types. Therefore, considering Table 1 also as a representation of possible reported types by each player, we observe that the first four rows in Table 1 represent reported type vectors wherein Player A is dishonestly reporting $s_A = 0.5$. The remaining rows represent type vector declaration such that Player A is reporting truthfully, or $s_A = \theta_A$. The corresponding BIC constraint has the following form:

$$\begin{aligned}
&\tfrac{0.7}{1.7}(0.1)(0.5)+\tfrac{0.7}{1.9}(0.1)(0.5)+\tfrac{0.7}{1.9}(0.9)(0.5)+\tfrac{0.7}{2.1}(0.9)(0.5) \\
&\qquad \leq (0.1)(0.5)[(1-p_{0.5})(1-p_{0.7})(\tfrac{0.5}{1.5})+(1-p_{0.5})(p_{0.7})(\tfrac{0.5}{1.5})+ \\
&\qquad\quad (p_{0.5})(1-p_{0.7})(\tfrac{0.7}{1.7}+x_L)+(p_{0.5})(p_{0.7})(\tfrac{0.7}{1.7}+x_L)]+...+ \\
&\qquad\quad (0.9)(0.5)[(1-p_{0.5})(1-p_{0.7})(\tfrac{0.5}{1.9})+(1-p_{0.5})(p_{0.7})(\tfrac{0.5}{1.9})+ \\
&\qquad\quad (p_{0.5})(1-p_{0.7})(\tfrac{0.7}{2.1}+x_L)+(p_{0.5})(p_{0.7})(\tfrac{0.7}{2.1}+x_L)], \qquad (12)
\end{aligned}$$

which simplifies to

$$\frac{573}{1615} \leq \frac{1367}{4845}+\frac{352}{4845}p_{0.5}+p_{0.5}x_L \qquad (13)$$

The left hand side of inequality (12) sums over the the last four rows in Table 1 and represents declarations wherein all three countries truthfully reveal their types. For this reason, inspection probabilities and penalties are irrelevant. No penalties will be assessed even if an inspection occurs because all agents are acting honestly. The only computation is the expected value of Player A's contribution by multiplying his contribution in each fully truthful declaration vector by the probability the truthful vector occurs.

The right hand side of the inequality considers scenarios wherein Player A acts dishonestly. In each term, the probability of a truthful vector occurring is multiplied by the product of the cost and penalty corresponding to a deceitful declaration and the probability of the corresponding inspection vector that induces it. For example, the first term considers Players B and C acting honestly by bidding {0.5, 0.5}, but Player A underbidding at $s_A = \{0.5\}$. The last term considers a similar situation, but with Players B and C having true types of {0.7, 0.7}. With two possible types in Φ,

there exist four inspection possibilities: no inspection, inspect $\{0.5\}$ declarations only, inspect $\{0.7\}$ declarations only, and inspect both declarations. The probability of each of these scenarios occurring is determined by our decision variables, $p_{0.5}$ and $p_{0.7}$. It can be observed that the probability of each of the inspection scenarios occurring is multiplied by the adjusted payment modified by the penalty and subsidy function, as defined in equation (3).

Following in this manner, it can be shown that the number of constraints required to enforce BIC is equal to $m\|\Phi\|\left(\|\Phi\|-1\right)$. The logic for this quantity of constraints is that a given player must be better off declaring his true type when considering the possibility of choosing any of the other $(\|\Phi\|-1)$ types. Thus, for a given player and a given true type, there are $(\|\Phi\|-1)$ constraints. As the given player has a positive probability of having any true type of the $\|\Phi\|$ types available, the number of constraints for a given player is $\|\Phi\|\left(\|\Phi\|-1\right)$. To find the total number of BIC constraints in the set, we must then consider the total number of players to obtain $m\|\Phi\|\left(\|\Phi\|-1\right)$. This implies that, in our instance having three players and two types in Φ, the number of BIC constraints required is six, in addition to the $2\|\Phi\|$ probability constraints and the two penalty non-negativity constraints, resulting in 12 total constraints. Notice, these results only hold when all players are have a type space of the same cardinality. In the case of independent spaces, Φ_j, for each player j, the number of BIC constraints equals $\sum_{j=1}^{m}\|\Phi_j\|\left(\|\Phi_j\|-1\right)$

When examining the SP formulation, a similar logic can be utilized, but without the luxury of assuming other players in the declaration vector are acting truthfully, meaning that decision variables may exist on both sides of the inequality. We now consider the game wherein all players have a true type of 0.7. In order to be strategyproof, Player A must be indifferent to the actions of Players B and C. For each possible set of declarations submitted by Player B and C, the rational choice for

Player A must be to truthfully report his type. We consider each of the possible declarations of Players B and C in turn.

Case 1: Player B and C report truthfully 0.7

$$\begin{aligned}\tfrac{0.7}{2.1} \leq & (1-p_{0.5})(1-p_{0.7})(\tfrac{0.5}{1.9}) + (1-p_{0.5})(p_{0.7})(\tfrac{0.5}{1.9}) + \\ & (p_{0.5})(1-p_{0.7})(\tfrac{0.7}{2.1}+x_L) + (p_{0.5})(p_{0.7})(\tfrac{0.7}{2.1}+x_L)\end{aligned} \tag{14}$$

Case 2: Player B reports truthfully $\{0.7\}$ and Player C deceives

$$\begin{aligned}(1-p_{0.5})(1-p_{0.7})(\tfrac{0.7}{1.9}) + (1-p_{0.5})(p_{0.7})(\tfrac{0.7}{1.9}) + & \\ (p_{0.5})(1-p_{0.7})(\tfrac{0.7}{2.1}-\tfrac{x_L}{2}) + (p_{0.5})(p_{0.7})(\tfrac{0.7}{2.1}-\tfrac{x_L}{2}) & \\ \leq (1-p_{0.5})(1-p_{0.7})(\tfrac{0.5}{1.7}) + (1-p_{0.5})(p_{0.7})(\tfrac{0.5}{1.7}) + & \\ (p_{0.5})(1-p_{0.7})(\tfrac{0.7}{2.1}+x_L) + (p_{0.5})(p_{0.7})(\tfrac{0.7}{2.1}+x_L) & \end{aligned} \tag{15}$$

Case 3: Player B deceives and Player C reports truthfully 0.7

$$\begin{aligned}(1-p_{0.5})(1-p_{0.7})(\tfrac{0.7}{1.9}) + (1-p_{0.5})(p_{0.7})(\tfrac{0.7}{1.9}) + & \\ (p_{0.5})(1-p_{0.7})(\tfrac{0.7}{2.1}-\tfrac{x_L}{2}) + (p_{0.5})(p_{0.7})(\tfrac{0.7}{2.1}-\tfrac{x_L}{2}) & \\ \leq (1-p_{0.5})(1-p_{0.7})(\tfrac{0.5}{1.7}) + (1-p_{0.5})(p_{0.7})(\tfrac{0.5}{1.7}) + & \\ (p_{0.5})(1-p_{0.7})(\tfrac{0.7}{2.1}+x_L) + (p_{0.5})(p_{0.7})(\tfrac{0.7}{2.1}+x_L) & \end{aligned} \tag{16}$$

Case 4: Player B and C deceive

$$\begin{aligned}(1-p_{0.5})(1-p_{0.7})(\tfrac{0.7}{1.7}) + (1-p_{0.5})(p_{0.7})(\tfrac{0.7}{1.7}) + & \\ (p_{0.5})(1-p_{0.7})(\tfrac{0.7}{2.1}+2(x_L)) + (p_{0.5})(p_{0.7})(\tfrac{0.7}{2.1}-2(x_L)) & \\ \leq (1-p_{0.5})(1-p_{0.7})(\tfrac{0.5}{1.5}) + (1-p_{0.5})(p_{0.7})(\tfrac{0.5}{1.5}) + & \\ (p_{0.5})(1-p_{0.7})(\tfrac{0.7}{2.1}+x_L) + (p_{0.5})(p_{0.7})(\tfrac{0.7}{2.1}+x_L) & \end{aligned} \tag{17}$$

In this example, the constraints consider all possible combinations of Player B's and Player C's actions and demonstrate a scenario wherein there exists only one type of deceitful action (i.e., declaring 0.5 when Player A's true type is 0.7). In general, the number of deceitful actions will equal $(\|\Phi\| - 1)$. From these results we can observe that, for each true type vector, a given player has $(\|\Phi\| - 1)\|\Theta_{-j}\| = (\|\Phi\| - 1)\|\Phi\|^{(m-1)}$ constraints to require a truthful action. This results from the fact that, for a given true type vector, every possible deceitful action by a given player must be an inferior course of action when all combinations of the remaining players' declarations are considered. Thus, in order to define all constraints associated with a given player, we must consider all true type vectors resulting in $(\|\Phi\| - 1)\|\Theta_{-j}\|\|\Theta\| = \|\Phi\|^{2m} - \|\Phi\|^{2m-1}$ constraints. Fortunately, in the current scenario having a common Φ available to each player, the constraints between players are symmetric. That is, should the true type vector be $\{0.5,0.5,0.5\}$, then the truthful constraint for Player A on the declared vector $\{0.7,0.5,0.5\}$ will be identical to Player B on the declared vector $\{0.5,0.7,0.5\}$ and Player C on $\{0.5,0.5,0.7\}$. It is sufficient in this symmetric situation to concern ourselves with only the constraints for a single player when solving the optimization problem. This implies that, in the example of three players and two types in Φ, the number of SP constraints required is 32 and the total for the problem instance is 38. However, in general, to define the total amount of SP constraints, we must consider all players, resulting in $\sum_{j=1}^{m}(\|\Phi_j\| - 1)\|\Theta_{-j}\|\|\Theta\|$ constraints where Φ_j is equal to the number of declarations available to the jth player.

The GSP formulation can be considered as a generalization of the SP formulation wherein the SP formulation only considers the singleton set of colluding players. It is well known from Pascal's triangle that the sum of the number of combinations across all subset sizes is equal to 2^n. When considering the size of $\mathcal{C}$, we exclude the empty set and observe the cardinality of $\mathcal{C}$ is equal to $2^n - 1$. There-

fore, in general, the GSP formulation sums over all $2^n - 1$ terms and has the form $\sum_{\mathcal{C} \subseteq \mathcal{M}}((\prod_{j \in \mathcal{C}} \|\Phi_j\|) - 1)\|\Theta_{-\mathcal{C}}\|\|\Theta\|$. The first term in the summation represents the number of group dishonest actions available to the colluding set of players. This term is then multiplied by the number of different declared types available to the non-colluding players and then by the number of possible true types, respectively. However, when players draw from an identical type space, we can again leverage the problem's symmetry and consider any subset of $\mathcal{M}$ such that it contains exactly one element of size 1 through m. We call any such subset $\hat{\mathcal{M}}$ and its cardinality is simply m. Summing over each of these terms, the number of GSP constraints under symmetry is $\sum_{\mathcal{C} \in \mathcal{M}'}(\|\Phi\|^{\|\mathcal{C}\|} - 1)\|\Theta_{-\mathcal{C}}\|\|\Theta\|$. As with SP under symmetry, we do not need to consider each colluding set individually. We only need to concern ourselves with the benefit a colluding set of a given size can achieve. Thus, the reduction of combinations from $2^n - 1$ to m serves to decrease the size of the GSP constraints dramatically for large instances having symmetry. In fact, the number of GSP constraints required for the example problem is 136.

IV. Results

The findings of the preceding chapter preclude the guarantee of gradient-based techniques finding an optimal solution to the BIC, SP or GSP problems. Thus, in this chapter, we explore the efficacy of several meta-heuristics and a global optimization solver with regard to these problems. We compare methods available in the MATLAB 2015a Optimization Toolbox, namely the *GlobalSearch*, *MultiStart*, and *ga* functions, utilizing the base settings to the global solver *BARON*. The *GlobalSearch* function implements the procedure introduced by Ugray *et al.* [2007] wherein a scatter search algorithm generates potential starting points, executes a gradient-based search (i.e., *fmincon*) on a feasible starting point, and then iterates through the remaining points by deeming them good or poor candidates. An additional gradient-based search is performed on each of the points deemed to be good candidates, whereas the poor candidates are discarded. The *MultiStart* procedure performs a gradient-based search technique from each point in a collection of uniformly distributed solutions within the feasible region. For consistency, we specify the gradient-based search as *fmincon* for use within both the *GlobalSearch* and *MultiStart* metaheuristics. The *ga* function applies a genetic algorithm with base settings including an initial solution population from a uniform distribution, a parent selection function utilizing a uniform distribution, a uniform binary crossover technique and a Gaussian mutation function [The MathWorks, Inc., 2015]. Of the three MATLAB heuristics, *MultiStart* and *ga* are compatible with parallel processing, whereas *GlobalSearch* is not. *BARON* is a general purpose, global optimization solver for mixed integer nonlinear programs. It utilizes a branch and reduce technique wherein a convex relaxation is generated and solved, a feasible solution is recovered, and the feasible region is subsequently partitioned to generate subproblems. This process continues for subproblems, enhanced by both feasibility and optimality-based range reduction, until a global optimal solution

(within some small ε tolerance) is identified with finite computational effort. Herein, we invoke the BARON commercial solver using the default termination criteria of a 0.001 (0.1

Section 4.1 solves the previously introduced sample instance for all problem variants using each solver. Utilizing the PSIE objective function with equal weights, we first examine the performance of each solver in terms of solution quality and computational efficiency. We then examine the effects of weighting the two PSIE objectives differently to explore the Pareto front associated with these problems. In Section 4.2, we expound upon the formulation size discussion from the previous section and demonstrate memory limitations encountered when solving the BIC, SP and GSP problems. Particularly, we show the difficulty in implementing the SP and GSP for large values of m or $\|\Phi\|$. Finally, Section 4.3 tests the superlative solution method identified in Section 4.1 on larger instances of the BIC problem.

4.1 Small Instance Testing

Analysis of Solver Performance.

In this section, we solve a BIC, SP and GSP problem instance with $\mathcal{M} = \{A, B, C\}$, $\Phi = \{0.5, 0.7\}$, $\lambda_j = 1$ for all players in $\mathcal{M}$, and equal weights in the PSIE objective function. This problem instance, and all following problem instances, are constructed utilizing two 2.60GHz Intel Xeon processors and 192GB of RAM within MATLAB. Solver methods internal to MATLAB are solved on the same machine. However, when invoking *BARON*, these problems are converted into a GAMS output file and solved on the NEOS server hosted by the University of Wisconsin in Madison [Czyzyk, J., Mesnier, M. P., and Moré, J. J., 1998; Dolan, E, 2001; Gropp & Moré, 1997]. Each method is given an upper bound of one hour to solve this small instance example. Our results in terms of solution quality of the objective function and solution time in

Table 3. Small Instance Testing Results for BIC, SP and GSP

Problem	BARON*		MultiStart**		GlobalSearch***		ga**	
	Objective	Solution Time(sec)	Objective	Solution Time(sec)	Objective	Solution Time(sec)	Objective	Solution Time(sec)
BIC	0.4709	0.315	0.4709	22.195	0.4709	734.121	0.4711	85.192
SP	0.482	5.915	1.2549	98.973	2.473	1013.452	0.4942	115.438
GSP	0.482	58.597	0.494	104.30	0.494	1187.666	0.491	153.186

* Problems constructed in MATLAB, GAMS file generated in MATLAB and uploaded to NEOS server.
** MATLAB parallel processing capability enabled.
*** Incompatible with MATLAB parallel processing. Solver utilizes serial computation.

seconds are summarized in Table 3.

We do not explicitly differentiate between the time spent solving a problem instance and the time spent building it. Instead, the total processing time is aggregated into the metric called solution time. This is done for consistency. For example, while *BARON* only requires the constraint building function to be called once, *MultiStart* and *GlobalSearch* may call the original constraint building function multiple times throughout the algorithm. Furthermore, the parallel algorithms in MATLAB make the distinction between build and solve times difficult to ascertain. Therefore, we are comparing the total efficacy of the process from problem construction to optimization.

The results in Table 3 indicate *BARON* finds the best solution in the shortest amount of time for all of the problem instances. When evaluating the MATLAB metaheuristics, *ga* is the second best performing algorithm with a minimum and maximum objective gap to *BARON* of 0.04% and 2.53%. However, it requires a minimum three-fold increase and a maximum of 270-fold increase in time from *BARON* to arrive at its solution. *MultiStart* is the second best performing algorithm with respect to solution time, but it falls short of *ga* in terms of solution quality. *MultiStart* arrives at a minimum and maximum objective gap to *BARON* of 0.00% and 160.35% while requiring a minimum twofold and maximum 70-fold increase in solution time. *GlobalSearch* is the worst performer in both metrics. Its minimum and maximum objective function gap to *BARON* is 0.00% and 413.07% and requires a minimum of 17-fold and a maximum of 2331-fold increase in solution time. Of note, the minimum and maximum relative gaps for each metaheuristic compared to *BARON* occurred

Table 4. BARON Solution Time Breakout for Small Instance Testing

Problem	Build Time(sec)	Solve Time(sec)	Total Time(sec)
BIC	0.312	0.003	0.315
SP	5.911	0.004	5.915
GSP	58.590	0.007	58.597

when solving the BIC and SP problems, respectively.

We close this discussion on solver performance by examining the solution time required for *BARON* in more detail. While *BARON* clearly outperforms the other tested solvers, the reported decision variable values are not necessarily optimal. Testing performed by Neumaier *et al.* [2005] demonstrated an experiment wherein *BARON* arrived at an incorrectly reported optimal solution for 1.8% of instances in an examined set. Also, it can be observed in Table 4 that the majority of *BARON's* solution time required does not result from the solution time, but from the time required to construct the instances. These results motivate further examination in subsequent sections on the computational effort required to create large instances of the SP and GSP problems.

Analysis of Decision Variables for Varying Truthfulness.

Examining the reported solutions by *BARON* and *ga* in Table 5 illustrates that, in this instance, the required increase in inspection probability and penalty values appear marginal compared to the increase in the strength of truthfulness constraints from BIC to GSP. However, in the arena of international alliances, consider the following: in December 2015, the UN's Fifth (Administrative and Budgetary) Committee recommended the General Assembly adopt a $5.4 billion budget for 2016-2017 [United Nations, 2015]. Should our example alliance of Countries A, B and C require its members to contribute this amount and adopt the solutions from BARON in Table 5, a BIC mechanism would require x_L = $1.068 billion, and a SP or GSP mechanism

Table 5. Reported Optimal Decision Variables for Small Instance Testing

	BARON				ga			
Problem	$p_{0.5}$	$p_{0.7}$	x_L	x_H	$p_{0.5}$	$p_{0.7}$	x_L	x_H
BIC	0.273	0.00	0.1979	0.00	0.266	0.00	0.2051	0.00
SP	0.280	0.00	0.202	0.00	0.2945	0.00	0.1997	0.00
GSP	0.280	0.00	0.202	0.00	0.301	0.00	0.190	0.00

would require x_L =\$1.091 billion, an increase of \$21.6 million (2.07%), in addition to the 0.007 increase in inspection probability, for a stronger guarantee of truthfulness. For many countries in the United Nations, this is a small increase when compared to their GDP. For example, the World Bank estimates the GDP of the United States to be \$17.95 trillion and the GDP of Greece to be \$195.21 billion. For other countries, such a fine represents a more substantial fraction of their GDP (i.e., Mongolia and El Salvador with respective GDP estimates of \$11.75 billion and \$25.85 billion). The World Bank also estimates another UN nation, Tuvalu, to have a \$37.75 million GDP. The Tuvalu economy would struggle to bear the burden of the increase in penalties alone imposed via the BIC and SP mechanisms. Therefore, as a central authority of the international alliance, the damage attributed to a country required to increase the expectation of truthfulness is relative, and alternative solutions attained by varying weights on the objectives may be necessary.

For this reason, we use *BARON* to estimate the Pareto front associated with these problem instances by applying the PSIE method and varying w_P from 0.1 to 0.9 in increments of 0.1, such that $w_I = 1 - w_P$. Utilizing these results we begin to observe the trade-off between inspection frequency and the size of deception penalties in Figure 1 and Table 6. At one extreme, we see the convergence of solutions across all instances to an inspection effort of one and a null penalty as w_P approaches one. This is due to the central authority's desire to minimize the penalties applied, requiring it to inspect and correct all information received to induce truthfulness

Table 6. Pareto Front Point Estimates for Small BIC, SP and GSP Instances

		BIC		SP		GSP	
w_I	w_P	Penalty Sum	Inspection Effort	Penalty Sum	Inspection Effort	Penalty Sum	Inspection Effort
0.9	0.1	0.743	0.091	0.758	0.094	0.762	0.093
0.8	0.2	0.471	0.136	0.482	0.140	0.482	0.140
0.7	0.3	0.342	0.178	0.352	0.182	0.349	0.183
0.6	0.4	0.260	0.223	0.266	0.227	0.263	0.230
0.5	0.5	0.1979	0.273	0.202	0.28	0.202	0.282
0.4	0.6	0.148	0.334	0.150	0.343	0.150	0.343
0.3	0.7	0.104	0.416	0.105	0.428	0.105	0.428
0.2	0.8	0.062	0.545	0.062	0.56	0.062	0.558
0.1	0.9	0.017	0.818	0.015	0.848	0.015	0.839

among its members. At the opposite extreme, we have an asymptote where the sum of the inspection effort approaches zero as w_I approaches one. The central authority wishes to inspect as infrequently as possible but levies a heavy fine on a deceitful nation should it detect dishonesty. The results indicate, for this instance, the central authority would not be required to allocate much more effort for inspections or levy substantially higher penalties to attain truthfulness under stronger assumptions (i.e., comparing BIC to SP or GSP). In fact, the SP and GSP curves are nearly on top of one another.

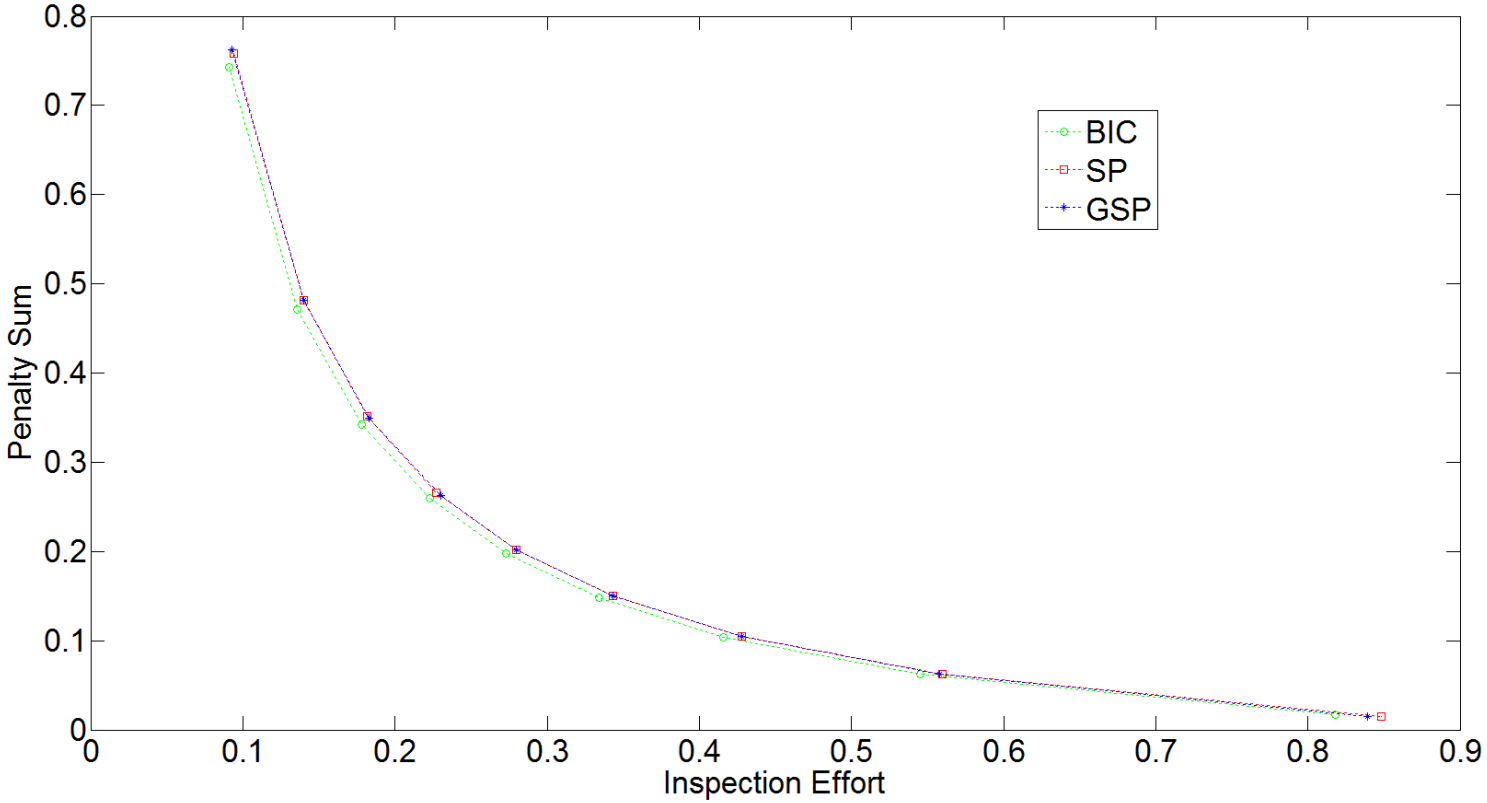


Figure 1. Pareto Front Estimates of BIC, SP and GSP Small Instances

This problem instance raises a question regarding the relevance of the GSP formulation. The solutions to all of the aforementioned SP and GSP instances, even with varying weights, are similar. We note this phenomenon does not hold for the general case, but it is a product of the penalty and subsidy function introduced in this research. In the aforementioned instances, the optimal SP decision variables limit the amount of potential subsidy in the system to such a degree, that little if any additional effort is required to prevent collusion. Thus, some of the optimal solutions to the SP instances are group strategyproof. However, the feasible regions of SP and GSP remain distinct, even if the optimal solutions nearly coincide. In fact, the optimal SP decision variables with $w_I = 0.9$ are infeasible in the corresponding GSP instance. In other words, the induced game is not group strategyproof.

Should additional subsidy be introduced, nations could experience more collusion benefit. Consider an alternative penalty and subsidy function wherein, if the central authority detects a deceitful action, it contributes $x_L + x_H$ of its own funds to be shared as a subsidy, in addition to the deception fines. Figure 2 shows the Pareto fronts associated with BIC, SP and GSP for the alternative penalty and subsidy function. In these GSP instances, additional inspection effort and/or deception penalties are required to deter collusion. Thus, the difference in effort between levels of truthfulness seems to be significantly influenced by the underlying penalties and subsidies used as incentive.

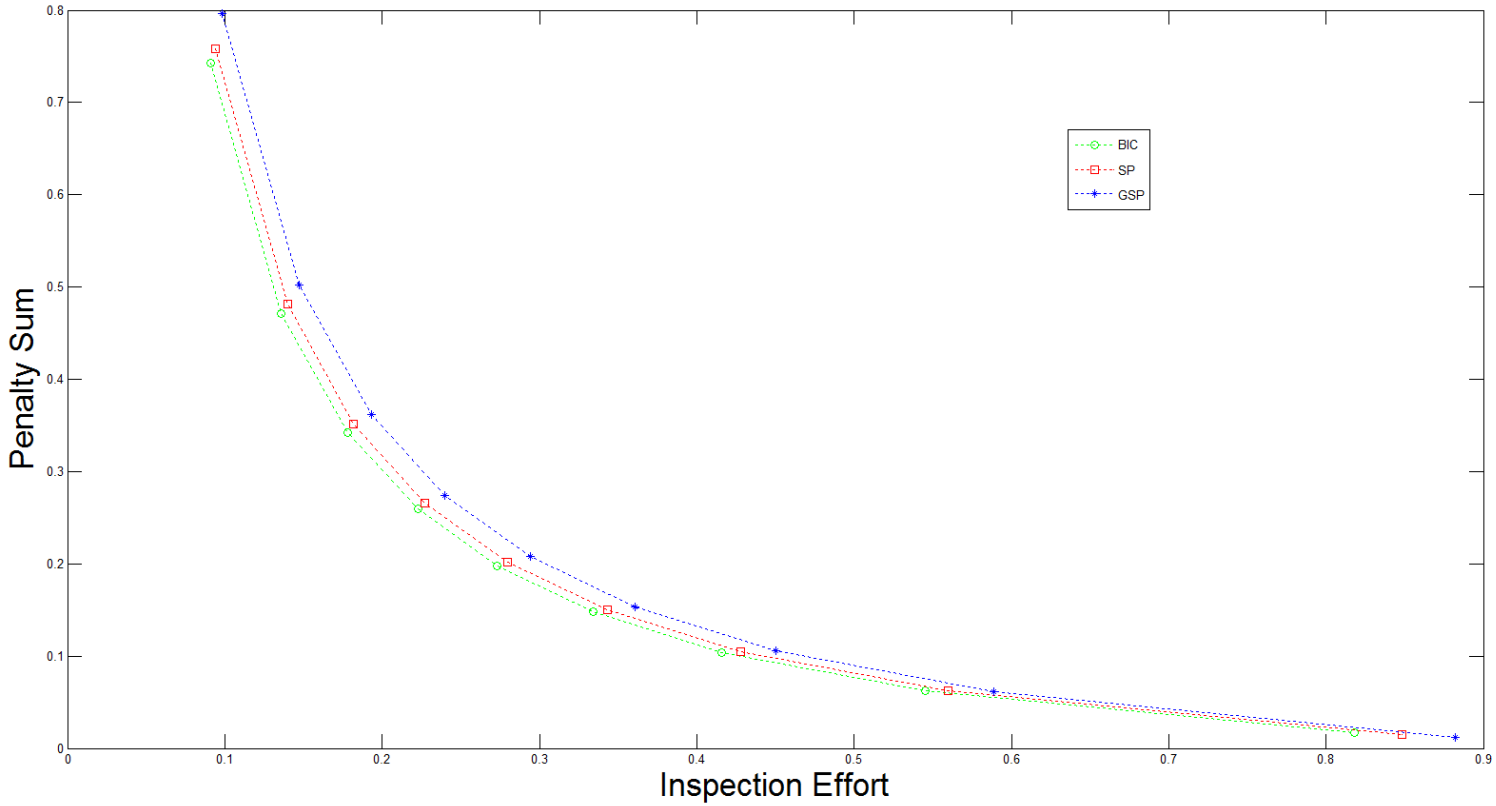
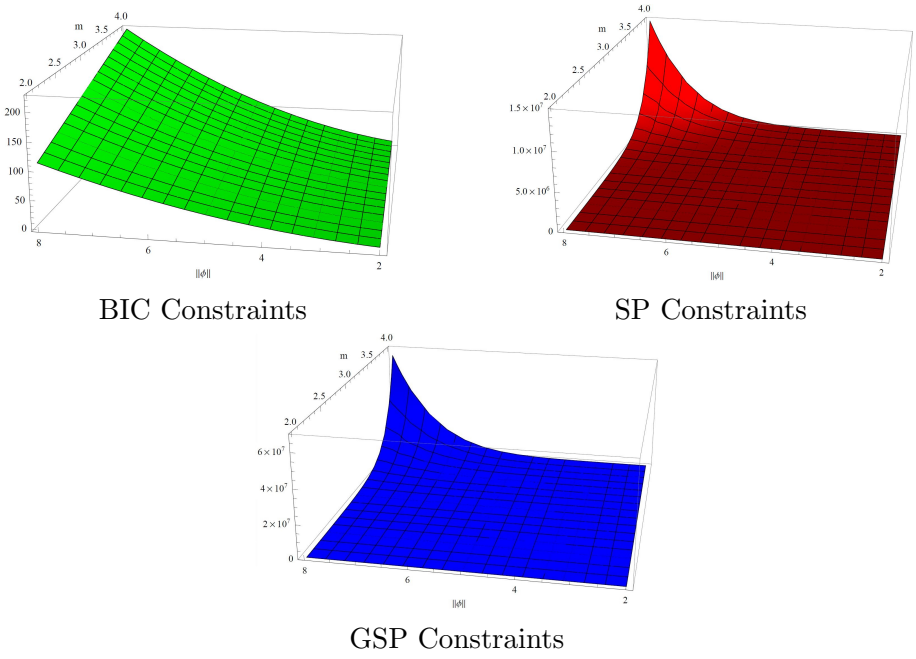


Figure 2. Pareto Front Estimates of BIC, SP and GSP Small Instances with Alternate Penalty and Subsidy Function

4.2 Memory Limitations

In Table 4, the build times required to construct problem instances and GAMS model files indicate a possible trend toward higher build times for larger instances. Graphically, we can visualize the dramatic increase in the number of constraints required for each formulation in Figure 3.



BIC Constraints

SP Constraints

GSP Constraints

Figure 3. Comparison of the Number of Constraints Required for Varying m and $\|\Phi\|$

From inspection of Figure 3, the difference in magnitudes of the number of constraints required for each formulation is immediately apparent. At $m = 4$ and $\|\Phi\| = 8$, the BIC, SP, and GSP problems necessitate 224, 1.468×10^7, 6.471×10^7 constraints, respectively, when it is assumed a common Φ is available to all players. Thus, it can be observed that memory limitations will become an obstacle sooner for GSP and SP than for BIC.

The machine utilized for the current study is equipped with 192GB of RAM. The ensuing discussion addresses an upper bound on problem size based on available RAM, independent of algorithm form or memory structures utilized to construct any

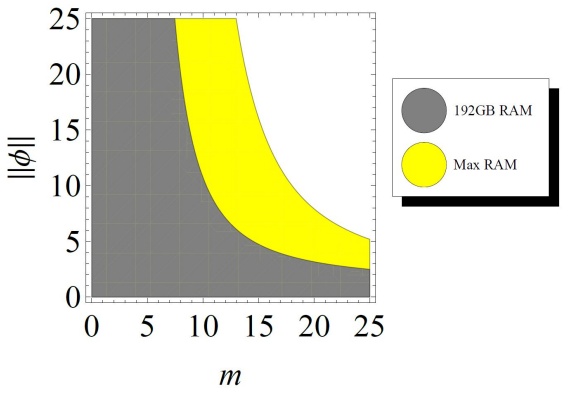


Figure 4. Feasible (m,$\|\Phi\|$)-combinations for a Computer with 192GB of RAM or 64 bit Theoretical Max (2^{34} GBs)

problem instance. For BIC, SP and GSP, should we assume that only Θ, the set of true type vectors, is stored in memory, the gray region under the lower curve in Figure 4 represents feasible Θ in MATLAB not exceeding 192GB of RAM. These calculations are based on the maximum array able to be stored in memory with $m\|\Theta\| = m\|\Phi\|^m$ double entries requiring eight bytes apiece.

By holding m constant, it can be observed that $\|\Phi\|$ has a polynomial effect on memory required, whereas m has an exponential effect for a fixed $\|\Phi\|$. Hence, in Figure 4, the memory limit is reached earlier by increasing m in isolation than by increasing $\|\Phi\|$. Considering the UN has 193 members, NATO 28 members, and the OAS 21 members, Figure 4 illustrates that meaningful instances for these alliances could not be be solved on the computer used in this study. Furthermore, a theoretical upper limit on RAM capable of being installed in a 64-bit computer can be derived from the largest unsigned integer possible (e.g., 2^{64} bytes) as this equals the number of addressable units. Thus, without concerning ourselves with operating system requirements, a maximum theoretical RAM limit for a 64 bit computer is 2^{34} GBs. In Figure 4, the yellow area under the higher curve represents Θ not exceeding the theoretical value of memory. We observe that a 25 nation coalition is limited to five declared types in Φ. An alliance the size of the UN would require so much memory that Φ

would be a singleton set. However, solving such an instance would be meaningless as it is impossible for any country to act deceitfully.

Having inspected the global memory limitations across all problem types, we now investigate the memory limitations inherent to the current study and the methodology utilized to generate each problem formulation. The predominant memory demands for all problems are composed of storing Θ, a matrix of equal dimensions as Θ listing all possibilities for each player of the cost sharing function, a matrix of possible inspection combinations, and a set of constraint strings. The matrix of inspection combinations is of type double with $2^{\|\Phi\|}$ entries and each constraint string is assumed to hold 56 characters requiring two bytes apiece. Figure 5 illustrates the feasible $(m,\|\Phi\|)$-combinations for instances we can formulate using 192GB of RAM and the aforementioned structures in memory for the BIC, SP and GSP problems, respectively. The blue portion of the graph represents feasible $(m,\|\Phi\|)$-combinations for all problem types. The red section indicates $(m,\|\Phi\|)$-combinations feasible for SP and BIC, while the green area indicates $(m,\|\Phi\|)$-values feasible only for BIC. Referencing Figures 4 and 5, we see the shape of the gray and green regions in each respective graph are very similar, implying that our formulation generation methodology is capable of creating an instance of BIC near the global upper bound of storing only Θ with 192GB of RAM. However, the additional memory structures required for SP and GSP become exceptionally burdensome and greatly limit the size of the alliance we can consider.

For this reason, the ensuing section of large instance problems will only consider the BIC formulation. The required data structures for SP and GSP cannot even be stored in memory to build such instances.

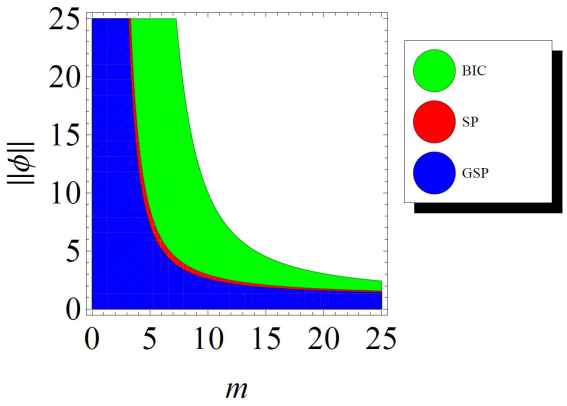


Figure 5. Feasible (m,$\|\Phi\|$)-combination Formulations by Problem Type with 192 GB RAM

4.3 Larger Instance Testing

Given the memory requirements for SP and GSP problems, we analyze the behavior of *BARON*-reported optimal solutions for larger instances of the BIC problem. However, the results of the previous section demonstrate that, for global alliances of substantial size, solving even the BIC problem becomes impractical. The larger instances herein examined are loosely based on UNASUR, a 12-nation alliance in South America analogous to the EU. Considering memory limitations, we set $\|\Phi\| = 5$. We again utilize the PSIE objective function with equal weights.

We conduct a full block design experiment with two factors: (1) the clustering of types in Φ, and (2) the composition of nation probability distributions over Φ. One possible interpretation of these factors is to consider them as the possible degree of wealthy disparity and the probability of existing a large amount of wealthy alliance nations in relative terms, respectively. The clustering factor is divided into five levels (i.e., five possible reserve price declarations), as described in Table 7. We consider alliances with a High End Cluster with Outlier (HEO), Low End Cluster with Outlier (LEO), High End Cluster (HEC), Low End Cluster (LEC), and Balanced (BAL) clustering of available types in Φ. The HEO and LEO levels represent situations

wherein a majority of available types lie close to one end of the spectrum, but an outlier exists near the opposite end. In HEO, the majority of available types represent a high valuation of the alliance with a single low valuation. LEO is the opposite representing a majority low valuation of the alliance, and a single high valuation. HEC and LEC are the same as HEO and LEO, but without the outliers. The final clustering factor, BAL, represents a scenario where valuations of the alliance are evenly spread between low and high valuations. These factors attempt to capture behavior exhibited from an alliance composed entirely or primarily of wealthy nations (HEC and HEO, respectively), entirely or primarily of less affluent nations (LEC and LEO, respectively), and of an even mixture of the two (BAL). Specifically, the HEO and LEO factors are designed to explore possibilities associated with behavior seen in NATO wherein a majority of the burden is born by a minority of the more affluent alliance members, as discussed in Olson & Zeckhauser [1966]. The second factor concerning the composition of nation probability distributions over Φ analyzes the effects of different types of uncertainty on the mechanism's parameters. We consider three levels of compositions as seen in Table 8. The first is an equitable population such that an identical number of players have a uniform, right skewed, left skewed, or symmetric distribution over Φ. The second and third compositions are either right skewed or left skewed wherein a plurality of players are predisposed to lower or higher valuations, respectively. The probabilities associated with each player distribution can be found in Table 8, and the levels of the population composition factor can be found in Table 9. Our full block design experiment will analyze all fifteen combinations of the five levels on the clustering factor and the three levels on the population composition.

Table 7. Larger Instance Testing: Categorical Factors on Clustering of Types in Φ

Clustering of Types in Φ	ϕ_1	ϕ_2	ϕ_3	ϕ_4	ϕ_5
High End Cluster with Outlier (HEO)	0.15	0.75	0.80	0.85	0.90
Low End Cluster with Outlier (LEO)	0.05	0.10	0.15	0.20	0.70
High End Cluster (HEC)	0.70	0.75	0.80	0.85	0.90
Low End Cluster (LEC)	0.05	0.10	0.15	0.20	0.25
Balanced (BAL)	0.10	0.30	0.50	0.70	0.90

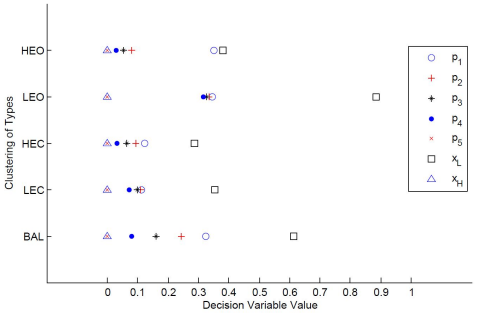
Table 8. Larger Instance Testing: Possible Nation Distributions Over Φ

Cluster of Types in Φ	P(ϕ_1)	P(ϕ_2)	P(ϕ_3)	P(ϕ_4)	P(ϕ_5)
Uniform (UNF)	0.20	0.20	0.20	0.20	0.20
Right Skewed Distribution (RS)	0.30	0.40	0.20	0.05	0.05
Left Skewed Distribution (LS)	0.05	0.05	0.20	0.4	0.30
Symmetric Distribution (SYM)	0.05	0.20	0.50	0.20	0.05

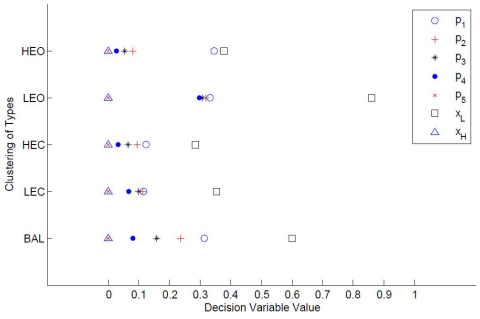
Table 9. Composition of Nation Distributions in Populations

Possible Distribution Types	UNF	RS	LS	SYM
Equitable Population	3	3	3	3
Right Skewed Population	0	6	3	3
Left Skewed Population	0	3	6	3

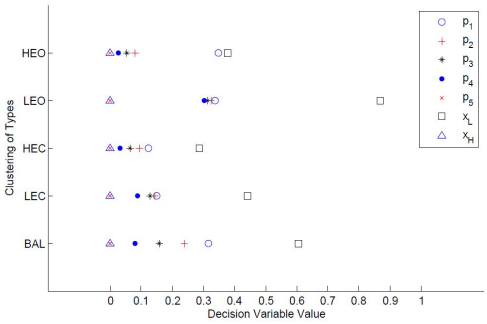
As with the small instance testing, the majority of solution time required for each instance is due to constraint generation. Each of the 15 instances required between five-and-a-half to six hours of build time, totaling approximately 88 hours of computation time. However, *BARON* required less than a single second to solve any given instance. The values for the decision variables of the experiments can be



Equitable Population



Right Skewed Population



Left Skewed Population

Figure 6. Decision Variable Results in the 12 Player and 5 Type Full Block Experiment

seen graphically in Figure 6, and higher fidelity information on the decision variables, objective function values, and computational effort required is available in Table 10.

Analyzing the objective function values with an equitable population composition across all levels of the clustering factors in Table 10, we see the pure low end cluster (LEC) and high end cluster (HEC) levels yield the lowest objective function values, whereas, the highest two objective function values are obtained from the LEO and BAL levels, respectively. This trend also holds under the right skewed population composition. In contrast, in the left skewed composition, HEC and HEO have the lowest objective function value, but LEO and BAL remain the highest. Considering each clustering individually, it can be observed that deception provides varying levels of benefit. In both the LEC and the HEC, a successful deceptive declaration can yield a maximum of 0.20 benefit, if a nation is of the highest type and dishonestly declares the lowest reserve price available (disregarding any potential subsidies from

other dishonest players). However, the same logic reveals that in the HEO and LEO the largest deceitful benefit is 0.65 and in BAL the value is 0.8. With these potential deception benefits in mind, the rationale behind the aforementioned objective function results begins to take form.

Across all clustering factors and population composition factor combinations, we observe inspection probabilities are decreasing across types in ascending rank order. That is, a type lower in value than another has a higher corresponding inspection probability. Inspection of the individual decision variable values and a knowledge of the underlying factors portends an explanation of *BARON's* solutions, which again relies upon the benefit of deception. For example, the LEC and HEC types are tightly clustered, yielding small potential deception benefits for nations of any true type, with a maximum of 0.20. The lower relative inspection probabilities of LEC and HEC across all population compositions are representative of the smaller benefits in deceit. Under LEO, a nation of true type ϕ_5 can benefit substantially from a successful deceptive bid of any of the other four types ranging from 0.50 to 0.65. Thus, we see high p_1 through p_4 values across all population compositions. Inherent in the structure of the type space, many different declarations have the ability to substantially reward deception in LEO or BAL. In either scenario, a player of type ϕ_5 has an array of declarations which would significantly reduce their contribution on a successful deceptive bid. For this reason, we observe these two clustering factors yield the highest penalties and generally high inspection probabilities. Similar analysis on HEO reveals a high probability of inspection on the lowest declarable reserve price and a large underbidding penalty is required to deter a player of a higher type to not declare ϕ_1.

Finally, several trends are visible with regard to solution times. Build times show a slightly increasing trend over population composition factors from equitable popula-

tions to right skewed populations and to left skewed populations. However, *BARON* solve times demonstrate notable differences over clustering factors. The LEO and HEO clustered problems require nearly a second to solve over all population compositions. HEC and BAL are comparable in requiring *BARON* times of approximately one third of a second. LEC uniformly requires the least amount of time, approximately one tenth of a second.

Based on these combined results, in general, the clustering factor is more significant than the population composition factor. This result is visible by inspection of Figure 6. All three population composition graphs look nearly identical, with the exception of the increased x_L value in the left skewed population when compared to the other two compositions. We also conjecture that optimal inspection probabilities are decreasing, or at least non-increasing, over types in ascending rank order. Finally, the elevated *BARON* solve times for HEO and LEO compared to the other instances leads us to conjecture these problems induce a higher degree of non-convexity.

Table 10. Decision Variable Values, Objective Function Values and Computational Effort in the 12 Player, 5 Type Block Experiment

	Symmetric Population					Right Skewed Population					Left Skewed Population				
Clustering	HEO	LEO	HEC	LEC	BAL	HEO	LEO	HEC	LEC	BAL	HEO	LEO	HEC	LEC	BAL
p_1	0.352	0.345	0.124	0.112	0.325	0.346	0.332	0.123	0.115	0.314	0.348	0.337	0.124	0.15	0.317
p_2	0.081	0.335	0.095	0.110	0.245	0.079	0.32	0.094	0.112	0.237	0.08	0.325	0.094	0.145	0.239
p_3	0.054	0.326	0.064	0.098	0.162	0.054	0.308	0.064	0.098	0.158	0.054	0.314	0.064	0.129	0.159
p_4	0.028	0.317	0.033	0.073	0.080	0.027	0.298	0.033	0.068	0.079	0.027	0.304	0.033	0.087	0.08
p_5	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
x_L	0.38	0.885	0.286	0.353	0.613	0.377	0.862	0.285	0.354	0.601	0.378	0.869	0.286	0.443	0.606
x_H	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
DV Sum	0.895	2.208	0.602	0.746	1.425	0.883	2.12	0.599	0.747	1.389	0.887	2.149	0.601	0.954	1.401
Build Time (sec)	21395.26	20664.14	20885.93	19437.85	18905.30	21547.73	20918.44	20995.44	20109.41	20528.56	21441.11	21204.22	21251.48	21210.87	21527.84
BARON Time (sec)	0.742	0.450	0.326	0.134	0.442	0.915	0.768	0.406	0.160	0.368	0.909	0.810	0.353	0.101	0.322

V. Conclusion

The declarable types in Φ are meant to represent the varying levels of worth a member state can place on an alliance. Since the types in Φ are the reserve prices of member nations in terms of the total proportion of alliance cost each nation is willing to contribute, differing definitions can be inferred on the meaning of worth. That is, a wealthy nation may feel ambivalent to the mission of an alliance but be willing to make an equitable contribution in proportion to the alliance cost. Similarly, a less affluent nation may feel strongly about the mission and choose to contribute a substantial amount. The implementation of a cost sharing mechanism for an international alliance is designed to elicit such preferences, limit the ability of a nation to exploit another, and reduce the risk of a single country disproportionately bearing part of the collective burden.

The current research examines optimization formulations having mechanisms that guarantee truthfulness among members by imposing constraints of variable strength. While any feasible solution to the BIC, SP, and GSP formulations correspond to games having a Bayes-Nash equilibrium, a Nash Equilibrium in dominant strategies, and a collusion resistant Nash equilibrium, respectively, an optimal solution to any of these problems yields an game based on the central authority's preferences. Each problem is non-convex and thus requires a global solver to find an optimal or near optimal solution. For very large alliances with many possible type declarations, all three of the problem variants are subject to memory limitations. However, it has been shown that the BIC problem is solvable in a reasonable amount of time for intermediate-sized instances. For such instances, the optimal solutions have been observed to be more sensitive to the clustering of types available for declarations than for the probability distributions over them.

5.1 Limitations

We note that some of the underlying assumptions of this research are impossible or improbable to occur in real world scenarios. Specifically, perfect inspection will not occur. Should an alliance assume a cost sharing mechanism as described, there would certainly be some error in discerning a nation's true type. Furthermore, the notion of a nation's true type is somewhat nebulous. A nation is not a single actor, but rather a collection of individuals. Thus, a single uniform valuation of an alliance in this context is unlikely. Instead, some variant of a social choice function would likely be required to ascertain an agreeable value for the nation. In practice, this value would likely not be voted on by the population but instead determined by a small number of national leaders.

Likewise, deception as described in this research is binary. However, we note that it is possible and, moreover, it is probable that Φ is composed of some highly dense set of values between zero and one. In such a situation, the mechanism described in this research would consider a bid very close, but still lower, than the true reserve price (e.g., one ten millionth), as a deceptive act. However, such a bid could easily be attributed to error from the nation or the central authority.

In general, a central authority ascertaining truthful preferences in any situation is dubious. Even if it were possible in an international alliance, inevitable political contention would ensue as the central authority attempted to collect the deception penalties, especially if the mechanism required overbidding penalties. If a central authority is not deemed credible by every country (i.e., some countries refuse to pay their penalties), the equilibrium results will not hold. The direct application of this research to an international alliance may not then be advisable. However, it is able to provide a bound on the effort required to ensure truthfulness in different equilibriums, and an understanding of what would be required to limit the effect

of dishonest political maneuver and coercion in an alliance. It also deviates from traditional burden sharing literature focusing on a central authority directly setting burden shares. Instead, the mechanisms in this research calculate the burden shares on the importance each nation places on the alliance.

5.2 Future Work

Quantifying the effects of probabilistic versus perfect inspection is a warranted sequel to this research. Likewise, given the memory limitations for analyzing very large alliances, there may exist a method to accurately aggregate an alliance's member nations into blocks such that a meaningful problem with respect to NATO or the UN is solvable. The underlying cost sharing function in this research is cross-monotonic. Alternative cross-monotonic cost sharing functions, or those which are not cross-monotonic, should be analyzed to determine if similar behavior is encountered.

If a cross sharing function that is not cross monotonic is selected, it may lose the property of being in the core, but it could be selected in such a way that it still exists in either the least core or the nucleolus. Should existence be in the least core, the allocation is such that the cost of deviating from the grand coalition, ε, has been reduced to the least feasible value. In contrast, existence in the nucleolus implies the maximum dissatisfaction of the grand coalition is minimized. Finally, the BIC problem assumes the independence of valuations by each member state. This property may be reasonable in general cost sharing circumstances; however, an international alliance may include countries with dependent types (e.g., the close historical relationship between the US and the UK). In such a scenario, the optimization problems as described are no longer appropriate and must be adapted to account for this interdependence.

Finally, the extension of this research to a corporate setting may allow for the direct application of the explored mechanisms. For this to occur, the central authority

must have enough power and credibility to enforce inspections and penalities. One such application may be the sharing of cost in a multi-party arbitration setting. However, it may be that the involved parties are no longer declaring reserve prices, but payment capacities. Other changes to the interpretations of the parameters utilized in this research would likely follow.

On Proportionate and Truthful International Alliance Contributions

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Sponsor:
USSTRATCOM

Objective

- Reformulate stable, proportional and truthful burden sharing mechanisms as a multiobjective optimization problem
- Understand the effort to maintain honesty as a best response under constraints of varying strength
- Categorize alliance sizes for which model can be solved
- Identify parameter effect on optimality

Modelling Framework

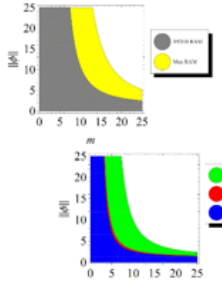
- Formulate three models whose solutions are yield proportionate burden sharing games in the core with a collectively honest strategy in equilibrium
  - Model 1: Bayesian Incentive Compatible
  - Model 2: Strategyproof
  - Model 3: Group Strategyproof

Related Literature

- Cost Sharing of a Public Good
- Burden Sharing
- Group Strategyproof Mechanisms
- Selected Heuristics

Burden Sharing Game

- Feasible reserve price & alliance size combinations based on memory limits



Math Programming Formulations

Bayesian Incentive (BIC) Compatibility

$$\min_{p,x_H,x_L} z = (f(p), h(x_H, x_L)) \quad (4a)$$

subject to

$$\sum_{\theta \in \Theta} \beta_{\theta,j} m(\theta_j \mid S_{-j}, \theta) \leq \sum_{\theta \in \Theta} \lambda_j \beta_{\theta,j} m(s_j \mid S_{-j}, \theta), \quad \forall j \in \mathcal{M}, s_j \neq \theta_j, \theta_j \in \Phi, \quad (4b)$$

(Every player; Each true type they can be; Every lie they can tell)

$$0 \leq p_{\phi_i} \leq 1, \quad \forall \phi_i \in \Phi, \quad (4c)$$

$$x_L, x_H \geq 0. \quad (4d)$$

Strategyproof (SP) Constraints

$$m(\theta_j \mid S_{-j}, \theta) \leq \lambda_j m(s_j \mid S_{-j}, \theta), \quad \forall j \in \mathcal{M}, S_{-j} \cup s_j \in \mathcal{S} : s_j \neq \theta_j, \theta \in \Theta \quad (7)$$

(Every player; Each true type vector; Every declaration vector where player j is dishonest)

Group Strategyproof (GSP) Constraints

$$\sum_{j \in \mathcal{C}} m(\theta_j \mid \hat{\theta}_{-j} \cup \hat{S}', \theta) \leq \sum_{j \in \mathcal{C}} \lambda_j m(\hat{s}_j \mid \hat{S}_{-j} \cup \hat{S}', \theta), \quad \forall \mathcal{C} \subseteq \mathcal{M}, \hat{S} \cup \hat{S}' \in \mathcal{S} : \hat{S} \neq \hat{\theta}, \theta \in \Theta. \quad (8)$$

(Every colluding set; Each true type vector; Every declaration vector where at least one colluding member is deceiving)

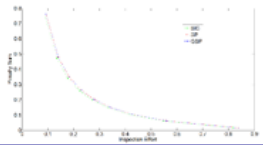
Identification of Superlative Solver Addressing Nonconvexity

Problem	BARON*		MultiStart**		GlobalSearch***		ga**	
	Objective	Solution Time(sec)	Objective	Solution Time(sec)	Objective	Solution Time(sec)	Objective	Solution Time(sec)
BIC	0.4709	0.315	0.4709	22.195	0.4709	734.121	0.4711	85.192
SP	0.482	5.915	1.2549	98.973	2.473	1013.452	0.4942	115.438
GSP	0.482	58.597	0.494	104.30	0.494	1187.666	0.491	153.186

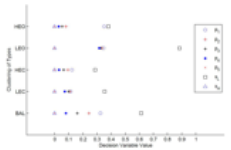
* Problems constructed in MATLAB, GAMS file generated in MATLAB and uploaded to NEOS server.
** MATLAB parallel processing capability enabled.
*** Incompatible with MATLAB parallel processing. Solver utilizes serial computation.

Analysis

- BARON shown to be effective solver
- Identify Pareto fronts for small instance of BIC, SP and GSP



- Alliances with large number of member nations or many available reserve prices will quickly encounter memory limits
- Explored effect of varying declarable reserve prices and set of country distributions thereover on optimality



Equitable Population

Conclusions

- SP and GSP instance sizes severely limited based on quantity of constraints
- Potentially little effort required by alliance to increase truthfulness under our penalty and subsidy function
- Clustering of reserve prices is significant

Future Work

- Consider the effect of placing nations into blocks to counter memory limits
- Effect of non-crossmonotonic cost sharing function
- Extension of proposed cost sharing mechanisms to the corporate setting

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14. ABSTRACT

Burden sharing within an international alliance is a contentious topic, especially in the current geopolitical environment, that in practice is generally imposed by a central authority's perception of its members' abilities to contribute. Instead, we propose a cost sharing mechanism such that burden shares are allocated to nations based on their honest declarations of the alliance's worth. We develop a set of mutliobjective nonlinear optimization problem formulations that, respectively, impose Bayesian Incentive Compatible (BIC), Strategyproof (SP) or Group Strategyproof (GSP) mechanisms based on probabilistic inspection and deception penalties that are budget balanced and in the core. A feasible solution to these problems produce a game wherein a collectively honest declaration is an equilibrium to the game but the optimal solution considers the central authority preferences. The efficacy of three heuristic algorithms and the BARON global solver are analyzed to determine the superlative methodology for each problem. The associated Pareto fronts are examined to determine the tradeoff between inspections and penalties required to obtain truthfulness under stronger assumptions. Memory limitations are examined to ascertain when the approach can be utilized. Finally, we consider the type clustering and nations' distributions therein on a UNASUR sized alliance.

15. SUBJECT TERMS

Burden Sharing, Cost Sharing, Group Strategyproof, Non-Convex Optimization, Mechanism Design

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